4.2. Logarithmic Functions

If x is a positive number, then the logarithm of x to the base $b (b > 0, b \neq 1)$, denoted $\log_b x$, is the number y such that $b^y = x$; that is,

$$y = \log_b x \quad \text{if and only if} \quad b^y = x \quad \text{for } x > 0$$

Example

Evaluate $\log_{10} 1,000$.

Example

Solve the equation $\log_4 x = \frac{1}{2}$.

Properties of Logarithms

Let $b (b > 0, b \neq 1)$ be any logarithmic base. Then,

$$\log_b 1 = 0 \quad \text{and} \quad \log_b b = 1$$

and if u and v are any positive numbers, then

- The equality rule: $\log_b u = \log_b v$ if and only if $u = v$
- The product rule: $\log_b(uv) = \log_b u + \log_b v$
- The power rule: $\log_b u^r = r \log_b u$ for any real number r
- The quotient rule: $\log_b \left(\frac{u}{v}\right) = \log_b u - \log_b v$
- The inversion rule: $\log_b b^u = u$

Properties of Logarithms

Example

Use logarithm rules to rewrite each of the following expressions in terms of $\log_3 2$ and $\log_3 5$.

a. $\log_3 270$

b. $\log_3 \left(\frac{64}{125} \right)$

Properties of Logarithms

Example

Use logarithm rules to simplify each of the following expression.

a. $\log_3(x^3 y^{-4})$

b. $\log_7(x^3\sqrt{1-y^2})$

The Natural Logarithm

The logarithm $\log_e x$ is called the natural logarithm of x and is denoted by $\ln x$; that is,

$$y = \ln x \quad \text{if and only if} \quad e^y = x$$

Properties of the Natural Logarithm

For positive numbers u and v,

- The equality rule: $\ln u = \ln v$ if and only if $u = v$
- The product rule: $\ln(uv) = \ln u + \ln v$
- The power rule: $\ln u^r = r \ln u$ for any real number r
- The quotient rule: $\ln\left(\frac{u}{v}\right) = \ln u - \ln v$
- Special values: $\ln 1 = 0$ and $\ln e = 1$

The Natural Logarithm

The Inverse Relationship between e^x and $\ln x$

$e^{\ln x} = x$ for $x > 0$ and $\ln e^x = x$ for all x

Example

Solve the following equations.

a. $-2 \ln x = 3$

b. $\ln x = 2(\ln 3 - \ln 5)$

c. $\dfrac{5}{1 + 2e^{-x}} = 3$

Conversion Formula for Logarithms

If a and b are positive numbers with $b \neq 1$, then

$$\log_b a = \frac{\ln a}{\ln b}$$

Example

Find $\log_5 3$.