

3.3. Curve Sketching

Vertical Asymptotes

The vertical line $x = c$ is a **vertical asymptote** of the graph of $f(x)$ if either

$$\lim_{x \rightarrow c^-} f(x) = +\infty \quad (\text{or } -\infty)$$

or

$$\lim_{x \rightarrow c^+} f(x) = +\infty \quad (\text{or } -\infty)$$

Vertical Asymptotes

Example

Determine all vertical asymptotes of the graph of

$$g(x) = \frac{2x^2 + 2x}{x^2 - 3x - 4}$$

Horizontal Asymptotes

The horizontal line $y = b$ is a **horizontal asymptote** of the graph of $f(x)$ if

$$\lim_{x \rightarrow -\infty} f(x) = b$$

or

$$\lim_{x \rightarrow +\infty} f(x) = b$$

Horizontal Asymptotes

Example

Determine all horizontal asymptotes of the graph of

$$g(x) = \frac{2x^2 + 2x}{x^2 - 3x - 4}$$

General Procedure for Sketching the Graph

- Step 1. Find the domain of $f(x)$.
- Step 2. Find and plot all intercepts.
- Step 3. Determine all vertical and horizontal asymptotes and draw them.
- Step 4. Find $f'(x)$ and determine the critical numbers and intervals of increase and decrease.
- Step 5. Determine all relative extrema. Plot each relative maximum with a “cap” and each relative minimum with a “cup”.
- Step 6. Find $f''(x)$ and determine intervals of concavity and points of inflection. Plot inflection points with a “twist”
- Step 7. Complete the sketch by joining the plotted points.

Curve Sketching

Example

Sketch the graph of $f(x) = \frac{4x}{(x+1)^2}$.

Curve Sketching

Example

Sketch the graph of $f(x) = \frac{x+3}{x-5}$.

Curve Sketching

Example

Sketch the graph of $f(x) = \frac{x + 1}{x^2 + x + 1}$.