2.5. Marginal Analysis and Approximation using Increments

Marginal Cost

If $C(x)$ is the total cost of producing x units of a commodity, then the *marginal cost* of producing x_0 units is the derivative $C'(x_0)$, which approximate the additional cost $C(x_0+1) - C(x_0)$ incurred when the level of production is increased by one unit, from x_0 to $x_0 + 1$.

Marginal Revenue and Marginal Profit

The *marginal revenue* is $R'(x_0)$. It approximates $R(x_0+1) - R(x_0)$, the additional revenue generated by producing one more unit.
The *marginal profit* is $P'(x_0)$. It approximates $P(x_0+1) - P(x_0)$, the additional profit obtained by producing one more unit.

Marginal Analysis

Example

$C(x) = \frac{1}{4}x^2 + 3x + 67$ is the total cost of producing x units and $p(x) = \frac{1}{5}(45 - x)$ is the price at which all x units will be sold.

(a) Find the marginal cost and the marginal revenue.

(b) Use marginal cost to estimate the cost of producing the fourth unit.

(c) Find the actual cost of producing the fourth unit.

(d) Use marginal revenue to estimate the revenue derived from the sale of the fourth unit.

(e) Find the actual revenue derived from the sale of the fourth unit.

Approximation by Increments

If $f(x)$ is differentiable at $x = x_0$ and Δx is a small change in x, then

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$$

or, equivalently, if $\Delta f = f(x_0 + \Delta x) - f(x_0)$, then

$$\Delta f \approx f'(x)\Delta x$$

Approximation by Increments

Example

A 5-year projection of population trends suggests that t years from now, the population of certain community will be $P(t) = -t^3 + 9t^2 + 48t + 200$ thousand.

(a) Find the rate of change of population $R(t) = P'(t)$ with respect to time t.

(b) At what rate does the population growth rate $R(t)$ change with respect to time?

(c) Use increments to estimate how much $R(t)$ changes during the first month of the fourth year. What is the actual change in $R(t)$ during this time period?

Approximation by Increments

Approximation formula for Percentage Change

If Δx is a (small) change in x, the corresponding percentage change in the function $f(x)$ is

$$\text{Percentage change in } f = 100\frac{\Delta f}{f(x)} \approx 100\frac{f'(x)\Delta x}{f(x)}$$

Example

Use increments to estimate the percentage change in the function $f(x) = 3x + \dfrac{2}{x}$ as x decreases from 5 to 4.6.