1.3. Linear Functions

Definition

A *linear function* is a function that changes at a constant rate with respect to its independent variable.

- The graph of a linear function is a straight line.
- The equation of a linear function can be written as

$$y = mx + b$$

where m and b are constants.

Linear Functions

Definition

The *slope* of the nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is the ratio

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Example

Find the slope of the line that passes through $(5, -1)$ and $(3, 3)$.

The Slope-Intercept Form

Definition (The Slope-Intercept Form)

The equation

$$y = mx + b$$

is the equation of the line whose slope is m and whose y-intercept is $(0, b)$.

Example

Find the equation of the line that passes through $(5, -1)$ and $(3, 3)$.

The Slope-Intercept Form

Example

Find the slope and y intercept of the line $5y - 3x = 4$.

Horizontal and Vertical Lines

Example

Find the equation of the line that passes through $(5, -1)$ and $(3, -1)$.

Example

Find the equation of the line that passes through $(5, -1)$ and $(5, 1)$.

The Point-Slope Form

Definition (The Point-Slope Form)

The equation

$$y - y_0 = m(x - x_0)$$

is the equation of the line that passes through the point (x_0, y_0) and that has slope equal to m.

Example

Find the equation of the line that passes through $(1, 2)$ with slope $\frac{2}{3}$.

The Point-Slope Form

Example

Find the equation of the line that passes through $(2, 5)$ and $(1, -2)$.

Practical Applications

Example

A certain car rental agency charges \$30 per day plus 55 cents per mile.

a. Express the cost of renting a car from this agency for 1 day as a function of the number of miles driven and draw the graph.

b. How much does it cost to rent a car for a 1-day trip of 250 miles?

c. How many miles were driven if the daily rental cost was \$74?

Parallel and Perpendicular lines

Let m_1 and m_2 be the slopes of the nonvertical lines L_1 and L_2. Then

- L_1 and L_2 are *parallel* if and only if $m_1 = m_2$.
- L_1 and L_2 are *perpendicular* if and only if $m_2 = \dfrac{-1}{m_1}$.

Example

Find the equation of the line that passes through $(-3, 2)$ and parallel to the line $x + 3y = 5$.

Parallel and Perpendicular lines

Example

Find the equation of the line that passes through $(1, 2)$ and perpendicular to the line $x + 3y = 5$.