1.2. The Graph of a Function

Definition

The *graph* of a function f consists of all points (x, y) where x is in the domain of f and $y = f(x)$; that is, all points of the form $(x, f(x))$.

Example

Graph the function $f(x) = x^2$.

x and y intercepts

Definition

The points (if any) where a graph crosses the x axis are called *x intercepts*, and a *y intercept* is a point where the graph crosses the y axis.

Example

Graph the function $f(x) = x^2 - 2x - 8$. Include all x and y intercepts.

x and y intercepts

Example

Graph the function

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 2 \\ x + 1 & \text{if } x \geq 2 \end{cases}$$

Include all x and y intercept.

Graphing parabolas

The graph of $y = Ax^2 + Bx + C$ is a parabola if $A \neq 0$.

- All parabolas have a U shape.
- The parabola $y = Ax^2 + Bx + C$ opens up if $A > 0$ and down if $A < 0$.
- The "peak" or "valley" of the parabola is called its *vertex*, and it occurs where $x = \frac{-B}{2A}$.

To get a reasonable sketch of the parabola, you need to determine

- The location of the vertex
- Whether the parabola opens up or down
- Any intercepts

Graphing parabolas

Example

Graph the function $f(x) = x^2 - 2x - 8$.

Manufacturing cost

Example

A bookstore can obtain an atlas from the publisher at a cost of \$10 per copy and estimates that if it sells the atlas for x dollars per copy, approximately $20(22 - x)$ copies will be sold each month. Express the bookstore's monthly profit from the sale of the atlas as a function of price, graph this function, and use the graph to estimate the optimal selling price.

Solution

- Let $x =$ price per atlas.
- Revenue generated each month will be

$$\begin{aligned} R(x) &= (\text{number of atlases sold}) \times (\text{price per atlas}) \\ &= [20(22 - x)][x] \end{aligned}$$

- Cost per month will be

$$\begin{aligned} C(x) &= (\text{number of atlases sold}) \times (\text{cost per atlas}) \\ &= [20(22 - x)][10] \end{aligned}$$

- Profit per month will be

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 20(22 - x)(x - 10) = -20x^2 + 640x - 4400 \end{aligned}$$

- By finding the vertex of this parabola, we see that $x = -B/2A = -640/-40 = 16$ maximizes profit.

Intersection of Graphs

Example

Find the points of intersection of the graphs of $y = x^2 - 1$ and $y = 3x + 3$.

Types of functions

Definition

A *power function* is a function of the form $f(x) = x^n$.

Example

$x^2, x^{-3}, x^{1/2}, \frac{1}{x^2}, \sqrt[3]{x}$

Definition

A *polynomial* is a function of the form
$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$.

Example

$p(x) = 3x^2 + 4x + 2, q(x) = x^5 + 2$

Definition

A quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$ is called a *rational function*.

Types of functions

Example

Classify the following functions.

a. $f(x) = -2 + 4x^2 + 3x^4$

b. $f(x) = \sqrt{x} + 5x$

c. $f(x) = \dfrac{(x-3)(x+7)}{-x^3 - 2x^2 + 3}$

d. $f(x) = \left(\dfrac{2x+7}{x^3-3}\right)^3$