1.1. Functions

Definition

- A *function* is a rule that assigns to each object in a set A exactly one object in a set B.
- The set A is called the *domain* of the function.
- The set of assigned objects in B is called the *range* of the function.

For the purposes of this class, the following will always be true.

- A and B will always be subsets of the real numbers $\mathbb{R}$.
- A function will be denoted by $f(x)$, and $f(x)$ will be given by a formula such as $f(x) = x^2 + 3$. Sometimes we write $y = f(x)$ where x is the *independent variable* and y is the *dependent variable*.
- The domain of $f(x)$ will be given explicitly (rarely) or will be the largest set of real numbers for which the formula for $f(x)$ makes sense (usually).

Example

Find $f(2)$ if $f(x) = x^2 + 3$.

Example

If $g(u) = (u+1)^{3/2}$, find $g(0), g(-1)$, and $g(8)$.

Piecewise-defined function

Example

Find $h(2), h(1), h(-2)$ if

$$h(x) = \begin{cases} -2x + 4 & \text{if } x \leq 1 \\ x^2 + 1 & \text{if } x > 1 \end{cases}$$

Examples: Finding the Domain

a. $f(t) = \dfrac{t+3}{t^2 - t - 2}$

b. $h(x) = \sqrt{x^2 - 4}$

Composition of functions

Definition

Given functions $f(u)$ and $g(x)$, the *composition* $f(g(x))$ is the function of x formed by substituting $u = g(x)$ for u in the formula for $f(u)$.

Example

Find the composite function $f(g(x))$, where $f(u) = u^2 + 3$ and $g(x) = x - 1$.

Composition of functions

Example

Find the composite functions $f(g(x))$ and $g(f(x))$, where $f(x) = x^2 + 3x + 1$ and $g(x) = 1 + x$. Note that $f(g(x)) \neq g(f(x))$.

Composition of functions

Example

At a certain factory, the total cost of manufacturing q units during the daily production run is $C(q) = q^2 + q + 900$ dollars. On a typical workday, $q(t) = 25t$ units are manufactured during the first t hours of a production run.

a. Express the total manufacturing cost as a function of t.

b. How much will have been spent on production by the end of the third hour?

c. When will the total manufacturing cost reach \$11,000?

Difference quotient

Definition

A *difference quotient* is an expression of the general form

$$\frac{f(x+h)-f(x)}{h}$$

where f is a function of x and h is a number.

Example

Find the difference quotient for $f(x) = 2x - x^2$.