Quiz 14 — 5.2

Final Exam — Monday 5/16

Review Session for final Tuesday 5/10

3 - 4^30 in this room.

$\int f(x)\,dx = F(x) + C$ where $F'(x) = f(x)$

Initial value problem: Find the value of C.

substitution as a "technique of integration"

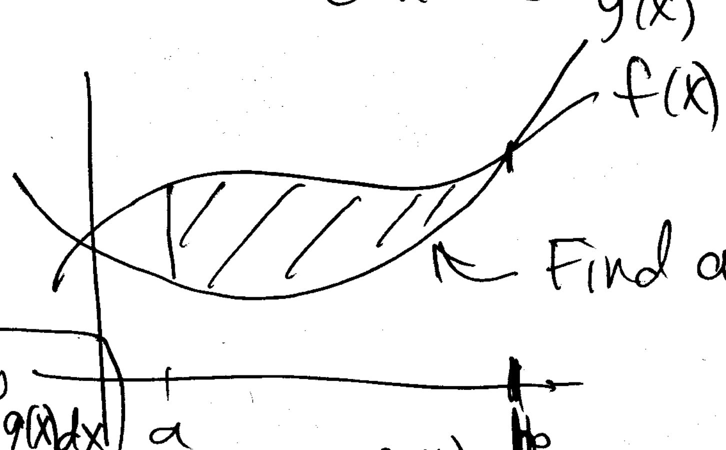
~~Fu~~ Fundamental Theorem of Calculus

Area under graph of f(x) for a≤x≤b. $= \int_a^b f(x)\,dx = F(x)\Big|_a^b = \underbrace{F(b) - F(a)}_{\text{total change in } F \text{ from } x=a \text{ to } x=b.}$

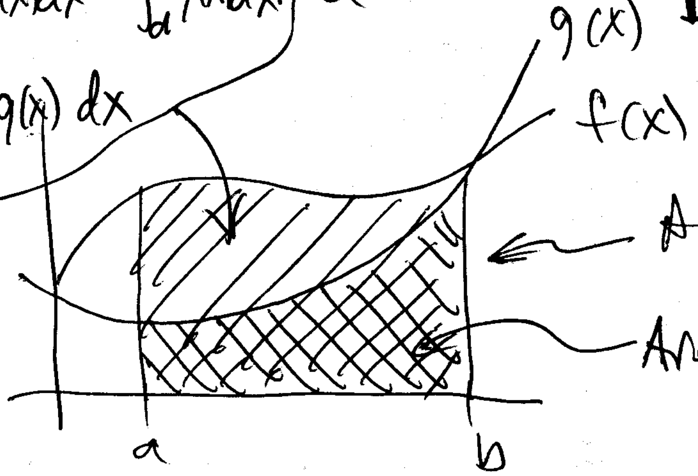
5.4 Applications of the Integral

A. Area between curves.

Idea:



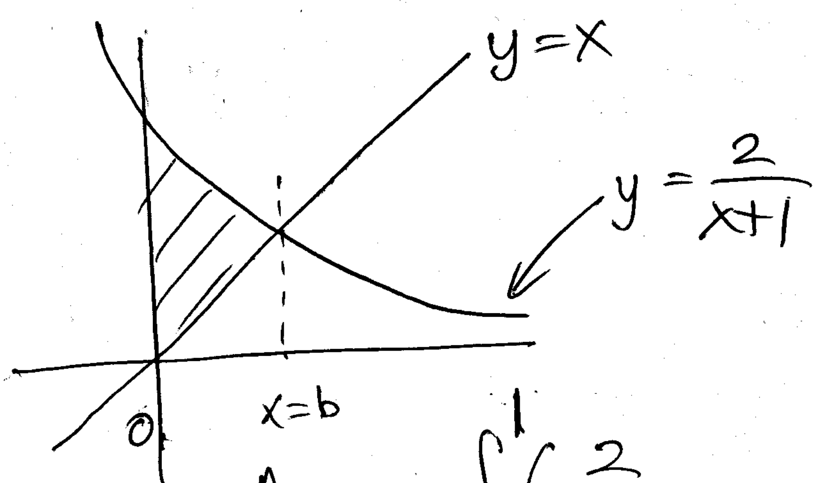
$$\boxed{\begin{aligned}\text{Area} &= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx \\ &= \int_a^b (f(x)-g(x))\,dx\end{aligned}}$$



$\text{Area} = \int_a^b f(x)\,dx$ ////

$\text{Area} = \int_a^b g(x)\,dx$ \\\\

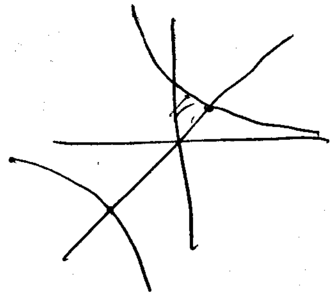
eg #3)



$$\text{Area} = \int_0^1 \left(\frac{2}{x+1} - x\right) dx$$

Need to find x=b where curves meet.

$$\frac{2}{x+1} = x$$



$$2 = x(x+1)$$

$$2 = x^2 + x$$

$$x^2 + x - 2 = 0$$

$$(x-1)(x+2) = 0$$

$$x = 1 \qquad \underline{x = -2}$$

$$A = \int_0^1 \left(\frac{2}{x+1} - x\right) dx$$

$$= 2\ln(x+1) - \frac{1}{2}x^2 \Big|_0^1$$

$$= 2\ln(2) - \frac{1}{2} - (2\overset{0}{\ln(1)} - 0)$$

$$= 2\ln(2) - \frac{1}{2}$$

$$\int \frac{2}{x+1}\, dx = \int \frac{2}{u}\, du$$

$u = x+1$

$du = dx$

$$= 2\ln(u) + C$$

$$= 2\ln(x+1) + C$$

$$\int x^r\, dx = \frac{1}{r+1} x^{r+1} + C$$

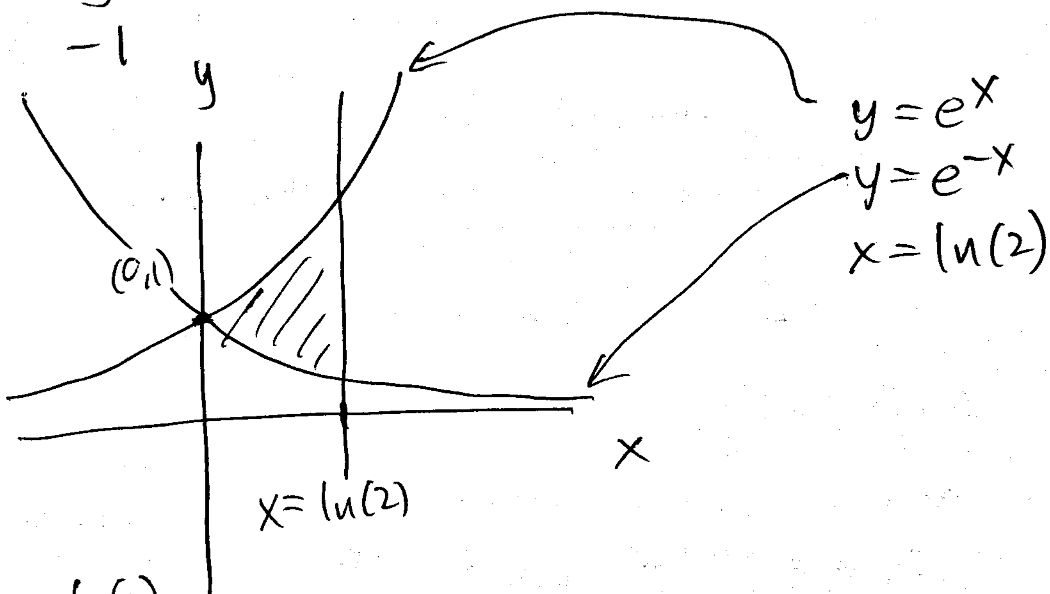
$$\int x^{-1}\, dx = \ln(x) + C$$

$$\int e^x\, dx = e^x + C$$

#4) $A = \int_{-1}^{2} (x^2+1) - (2x-2)\, dx$

$= \int_{-1}^{2} (x^2 - 2x + 3)\, dx$ - - - -

#8)



$A = \int_0^{\ln(2)} (e^x - e^{-x})\, dx$

$= e^x - (-e^{-x}) \Big|_0^{\ln(2)}$

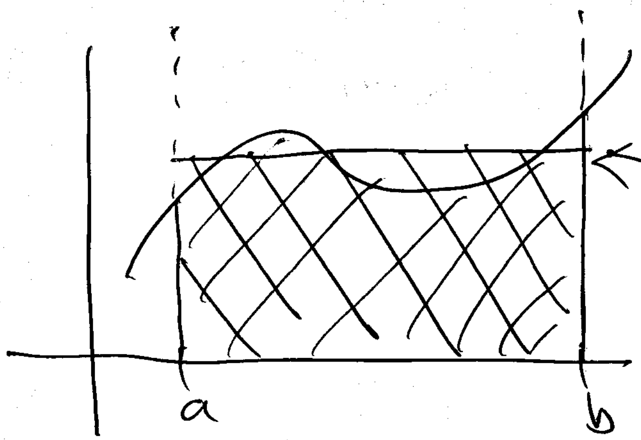
$e^{-\ln(2)} = \frac{1}{e^{\ln(2)}} = \frac{1}{2}$

$= e^x + e^{-x} \Big|_0^{\ln(2)}$

$= e^{\ln(2)} + e^{-\ln(2)} - (e^0 + e^0)$

$= 2 + \frac{1}{2} - 2 = \frac{1}{2}$ //

B. Average value.



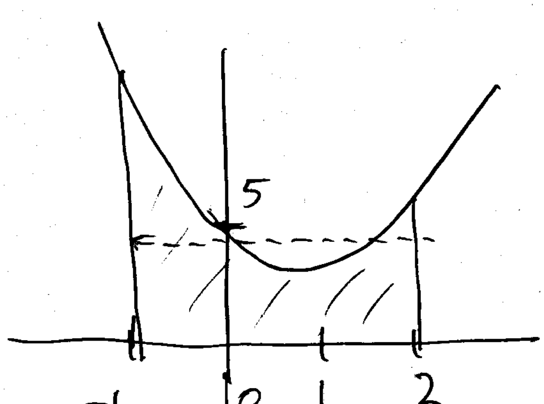
$$\text{Area} = \int_a^b f(x)\,dx$$

average value of $f(x)$ for $a \le x \le b$.

$$\int_a^b f(x)\,dx = \left(\begin{matrix}\text{Avg}\\ \text{value}\end{matrix}\right) \times (b-a)$$

$$\therefore \boxed{\begin{matrix}\text{Avg}\\ \text{value}\end{matrix} = \frac{1}{b-a}\int_a^b f(x)\,dx.}$$

#20) $f(x) = x^2 - 3x + 5 \qquad -1 \le x \le 2$



$$\begin{matrix}\text{Avg}\\ \text{value}\end{matrix} = \frac{1}{2-(-1)}\int_{-1}^{2}(x^2-3x+5)\,dx$$

$$= \frac{1}{3}\left(\frac{1}{3}x^3 - \frac{3}{2}x^2 + 5x \Big|_{-1}^{2}\right)$$

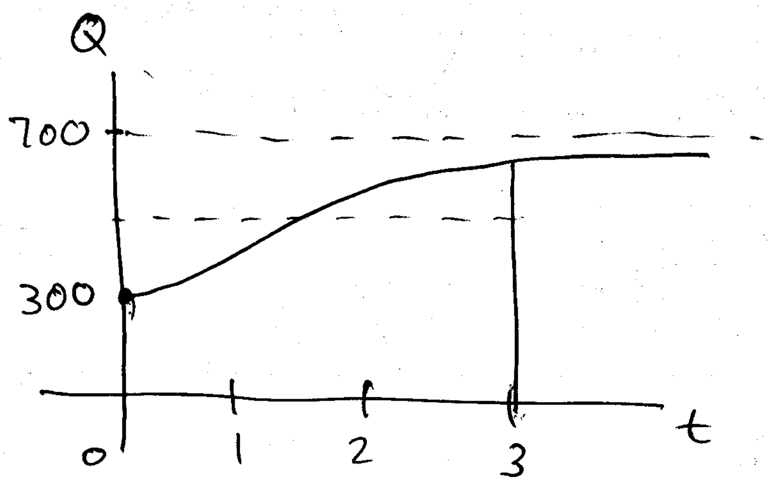
$$= \frac{1}{3}\left(\frac{8}{3} - 6 + 10 - \left(-\frac{1}{3} - \frac{3}{2} - 5\right)\right)$$

$$= \frac{1}{3}\left(\frac{8}{3} + 4 + \frac{1}{3} + \frac{3}{2} + 5\right) = \frac{1}{3}\left(12 + \frac{3}{2}\right),$$

$$= \frac{1}{\cancel{3}}\left(\frac{\overset{9}{\cancel{27}}}{2}\right) = \frac{9}{2}$$

#36) $Q(t) = 700 - 400e^{-.5t}$ letters/hour

Avg value of Q for $0 \le t \le 3$



$$\text{Avg value} = \frac{1}{3}\int_0^3 \left(700 - 400e^{-.5t}\right)dt$$

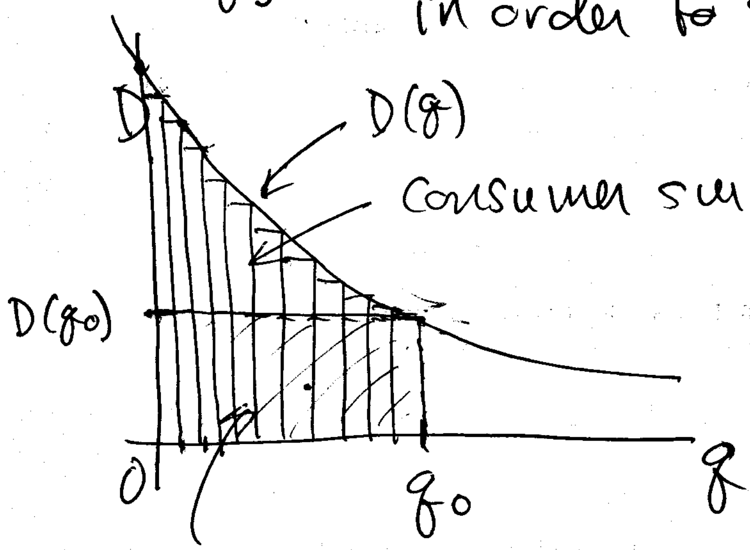
$$= \frac{1}{3}\left(700t - 400\left(\frac{1}{-.5}e^{-.5t}\right)\right)\Big|_0^3$$

$$= \frac{1}{3}\left(700t + 800e^{-.5t}\right)\Big|_0^3 \quad \text{etc}\ldots$$

5.5 Additional Apps.

A. Consumer surplus / willingness to spend.

$D(q)$ = price that must be charged in order to sell q units



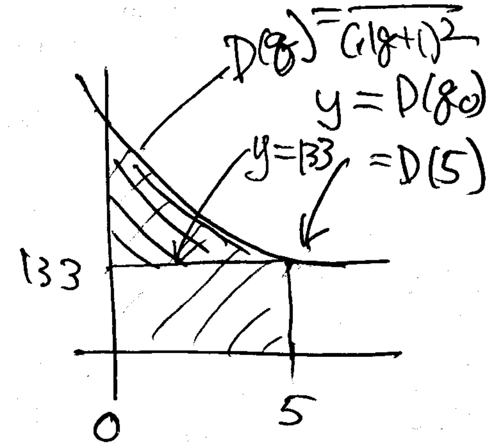
Total area = Consumers willingness to spend.

Area $= D(q_0) \cdot q_0 =$ revenue generated by q_0 units

eg #2) $D(q) = \dfrac{300}{(.1q+1)^2}$ $\quad q_0 = 5$

Consumers willingness to spend $= \displaystyle\int_0^5 \frac{300}{(.1q+1)^2}\,dq$

Consumer surplus:



$D(5) = \dfrac{300}{(.1(5)+1)^2} = \dfrac{300}{(\frac{3}{2})^2} = \dfrac{300 \cdot 4}{9}$

$\doteq 133$

$$\text{Surplus} = \int_0^5 \left(\frac{300}{(.1q+1)^2} - 133\right) dq$$

$$\int_0^5 \frac{300}{(.1q+1)^2}\, dq = -\frac{3000}{(.1q+1)}\Big|_0^5 = \frac{-3000}{\frac{3}{2}} - (-3000)$$

$$= -2000 + 3000$$

$$= 1000 \text{ dollars.}$$

$$\int \frac{300}{(.1q+1)^2}\, dq \qquad u = .1q+1$$

$$du = .1\, dq$$

$$10\, du = dq$$

$$= 300 \int \frac{1}{u^2} \cdot 10\, du$$

$$= 3000 \int \frac{1}{u^2}\, du = 3000(-u^{-1})$$

$$= -\frac{3000}{u} = -\frac{3000}{(.1q+1)}$$