Quiz 14 Wednesday 5/4 5.2

Final Exam - Monday 5/16.

cumulative

About half will be on Ch. 5.

About half on previous sections

Chapters 2 + 3 emphasized

5.3 Definite Integral + Fund. Thm. of Calc.

Example: $s(x)$ - position at time x

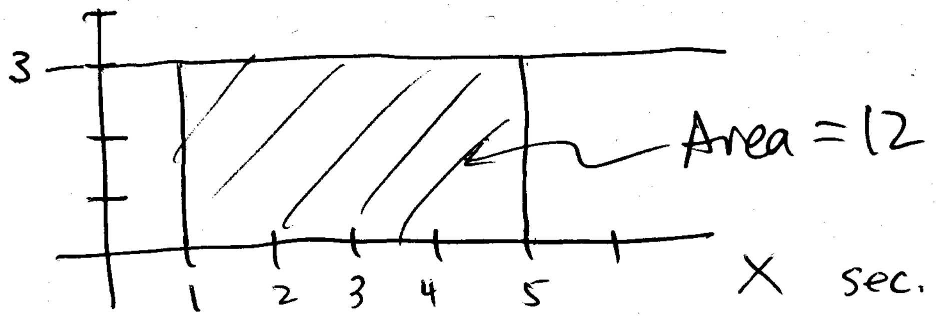
$s'(x)$ - velocity or speed.

Just knowing $s'(x)$ allows us to find our relative position or the distance we travel between two points of time

$$s(x) = \int s'(x)\,dx \qquad \int s'(x)\,dx = s(x) + C$$

Suppose $s'(x) = 3$ $\left(\frac{\text{meters}}{\text{second}}\right)$

Graphically



Between $x=1$, $x=5$ how far have I travelled?

$D = 3(5-1) = 3 \cdot 4 = 12$ m.

Another way to answer this:

$s'(x) = 3$

$s(x) = 3x + C$

Distance travelled $= s(5) - s(1)$

$= (3(5) + \cancel{C}) - (3(1) + \cancel{C})$

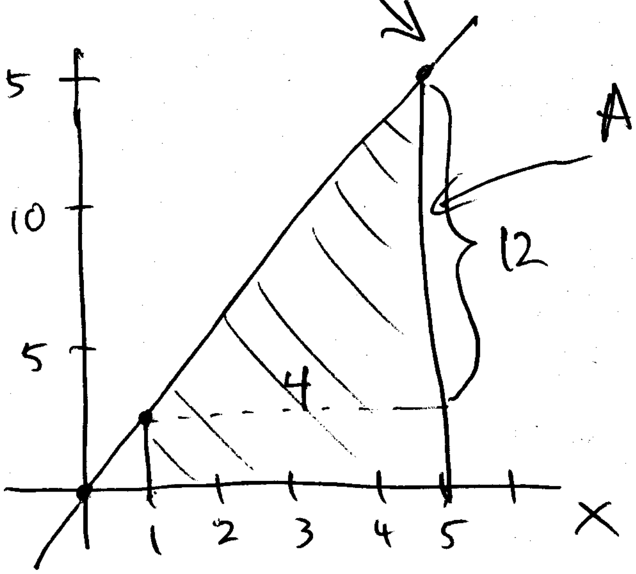
$= 3(5) - 3(1) = 3(5-1) = 12.$

We have a connection between

Area under the graph of $s'(x)$ **AND** Displacement of $s(x)$ between 2 values of x.

$s'(x) = 3x$

constant acceleration



$$\text{Area} = \frac{1}{2}(4)(12) + (4)(3) = 24 + 12 = 36$$

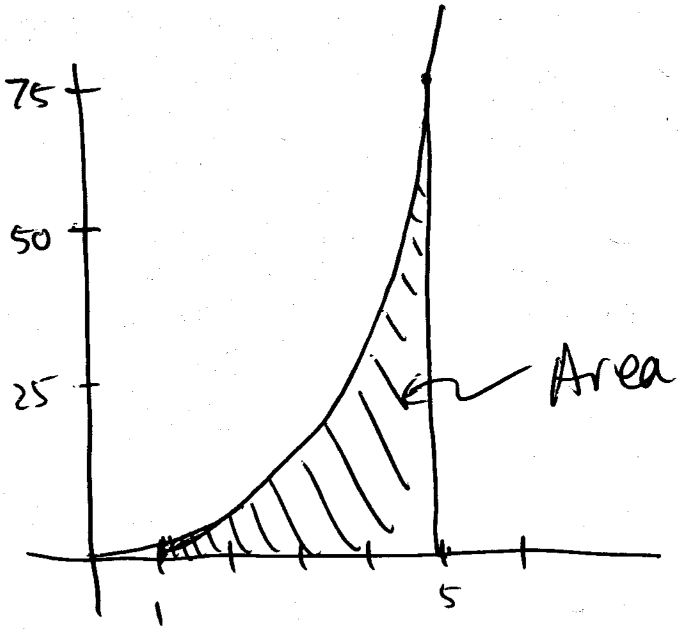
$$s(x) = \frac{3}{2}x^2 + C$$

$$\text{Dist. travelled} = s(5) - s(1)$$

$$= \left(\frac{3}{2}(5)^2 + \cancel{C}\right) - \left(\frac{3}{2}(1)^2 + \cancel{C}\right)$$

$$= \frac{3}{2}\cdot 25 - \frac{3}{2}\cdot 1 = \frac{3}{2}(24) = 36 \text{ m.}$$

$s'(x) = 3x^2$



Area = ?

Is the area also 124?

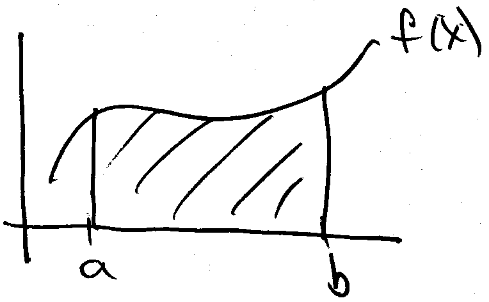
Total distance travelled between $x=1$, $x=5$:

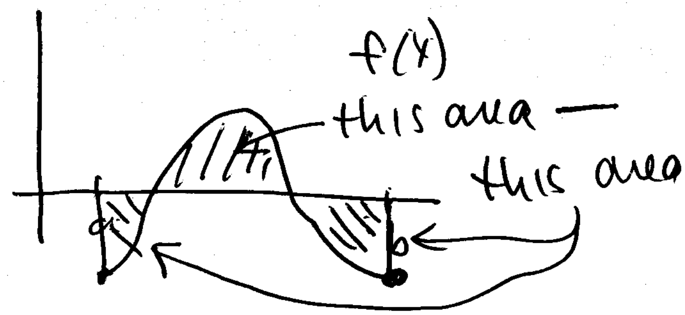
~~Distan~~ $s(x) = x^3 + c$

Distance $= s(5) - s(1)$

$= (5^3 + \cancel{c}) - (1^3 + \cancel{c})$

$= 125 - 1 = 124$ m





The Definite Integral

The area under the graph of f(x) between x=a, x=b.

The Fundamental Theorem of Calculus

If the function $f(x)$ is continuous on the interval $a \leq x \leq b$, then

$$\boxed{\int_a^b f(x)\,dx} = F(x)\Big|_a^b = F(b) - F(a)$$

$F'(x) = f(x)$

where $F(x)$ is any antiderivative of $f(x)$ on $a \leq x \leq b$.

can also write

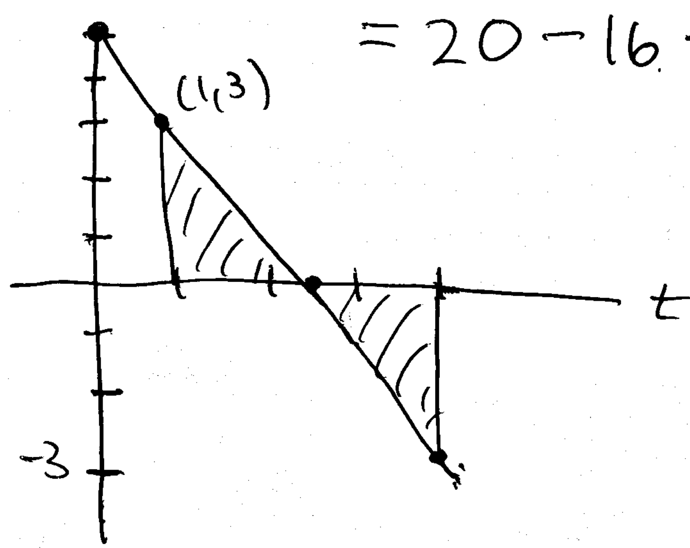
$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Example (#4)

Evaluate $\int_1^4 (5 - 2t)\,dt$.

$$= 5t - t^2 \Big|_1^4 = (5(4) - (4)^2) - (5(1) - (1)^2)$$

$$= 20 - 16 - 5 + 1 = 0$$



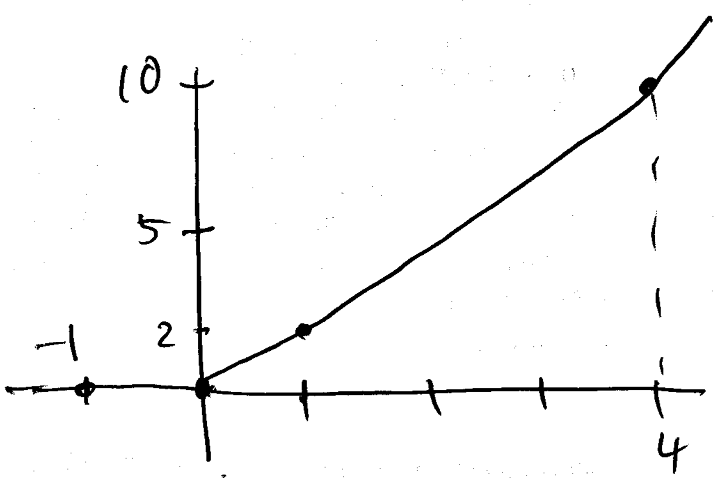
The Definite Integral

Area as a Definite Integral

If $f(x)$ is continuous and $f \geq 0$ on the interval $a \leq x \leq b$, then the region under the curve $y = f(x)$ over the interval $a \leq x \leq b$ has area given by the definite integral $\int_a^b f(x)\,dx$.

Example (#38)

Find the area of the region that lies under the curve $y = \sqrt{x}(x+1)$ over the interval $0 \leq x \leq 4$.



$$A = \int_0^4 \sqrt{x}(x+1)\,dx = \int_0^4 x^{1/2}(x+1)\,dx = \int_0^4 (x^{3/2} + x^{1/2})\,dx$$

$$= \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{2}{5}(4)^{5/2} + \frac{2}{3}(4)^{3/2} - 0$$

Rules of Definite Integrals

Let f and g be continuous on $a \leq x \leq b$. Then

- The constant multiple rule:

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx \quad \text{for constant } k$$

- The sum rule:

$$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

- The difference rule:

$$\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Rules of Definite Integrals

Let f and g be continuous on $a \leq x \leq b$. Then

- $\displaystyle\int_a^a f(x)\,dx = 0$
- $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
- The subdivision rule:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Rules of Definite Integrals

Example (#36)
Let $f(x)$ and $g(x)$ be continuous on $-3 \leq x \leq 1$ and satisfy

$$\int_{-3}^{1} f(x)\, dx = 0 \quad \int_{-3}^{1} g(x)\, dx = 4$$

Evaluate $\int_{-3}^{1} [2f(x) + 3g(x)]\, dx$.

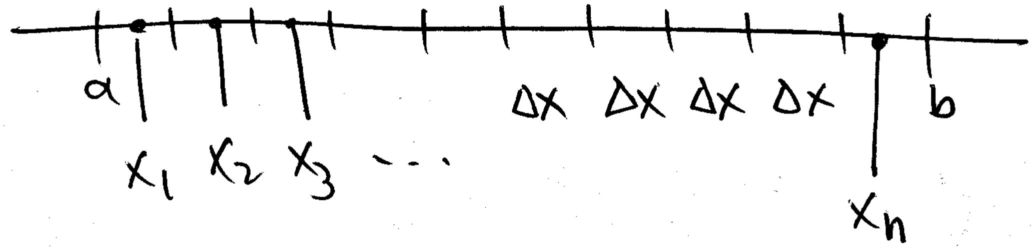
Rules of Definite Integrals

Example (#32)

Let $g(x)$ be continuous on $-3 \leq x \leq 2$ and satisfies

$$\int_{-3}^{2} g(x)\ dx = -2 \quad \int_{-3}^{1} g(x)\ dx = 4$$

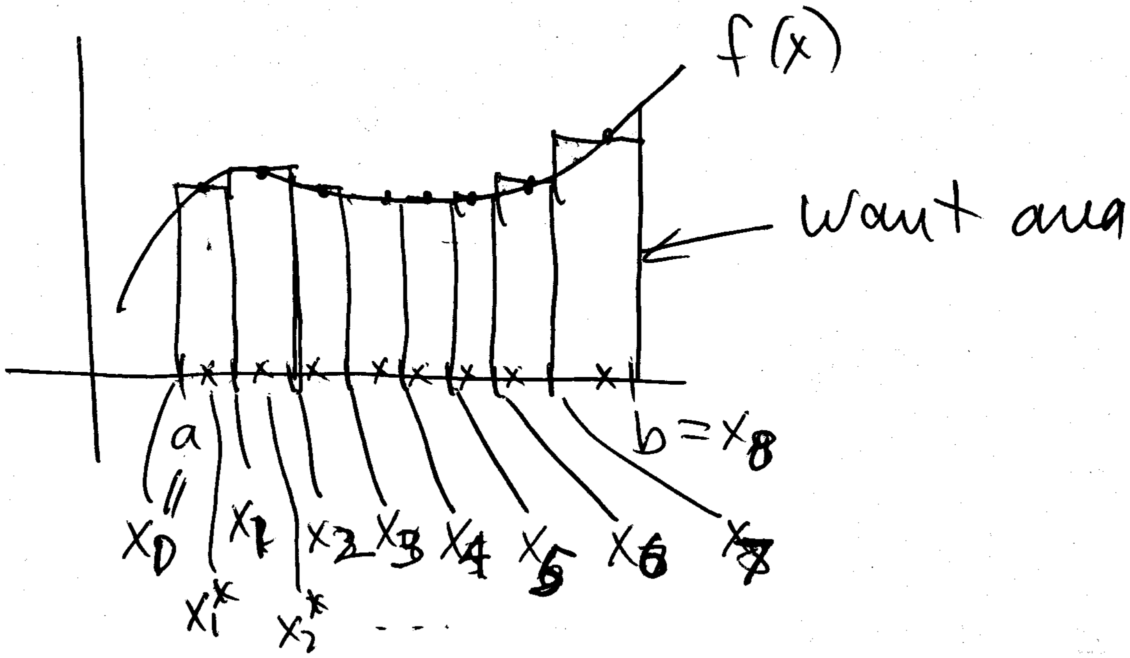
Evaluate $\int_{1}^{2} g(x)\ dx$.



$$\text{Area} \approx \underbrace{\left[f(x_1)+f(x_2)+\cdots+f(x_n)\right]}_{\text{sum}} \Delta x$$

$$= \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$\xrightarrow{\Delta x \to 0} \int_a^b f(x)\,dx$$



$$\text{Area} \approx (x_1 - x_0) f(x_1^*) + (x_2 - x_1) f(x_2^*) + \cdots + (x_8 - x_7) f(x_8^*)$$

Riemann sum.

Approximates area under curve.

$$\text{Exact area} = \lim \left(\text{Riemann sums}\right) = \int_a^b f(x)\,dx$$

width of intervals $\to 0$.

The Definite Integral

Let $f(x)$ be a continuous function on $a \leq x \leq b$. Subdivide the interval $a \leq x \leq b$ in n equal parts, each of width $\Delta x = \dfrac{b-a}{n}$, and choose a number x_k from the kth subinterval. Form the sum

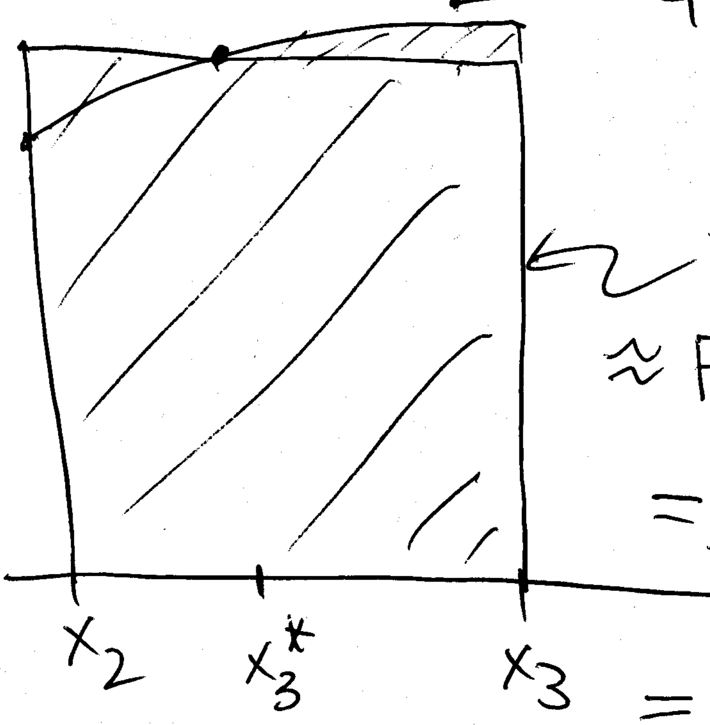
$$[f(x_1) + f(x_2) + \cdots + f(x_n)]\Delta x$$

called the Riemann Sum.
Then the definite integral of f on the interval $a \leq x \leq b$, denoted by $\displaystyle\int_a^b f(x)\ dx$, is the limit of the Riemann sum as $n \to +\infty$; that is,

$$\int_a^b f(x)\ dx = \lim_{n \to +\infty} [f(x_1) + f(x_2) + \cdots + f(x_n)]\Delta x$$

Take one rectangle and blow it up:

$f(x) = F'(x)$



Area of rect

$\approx F'(x_3^*)(x_3 - x_2)$

$= f(x_3^*)(x_3 - x_2)$

$= F(x_3) - F(x_2)$