108 Lecture notes 2/2/11

1.5

25) $$\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x \to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}$$

$$\frac{(\sqrt{x}-2)}{x-4} \cdot \frac{(\sqrt{x}+2)}{\sqrt{x}+2} = \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2}$$

Difference quotient : $\frac{f(x+h)-f(x)}{h}$

Idea: x = time (on a clock)

$f(x)$ = reading on trip odometer

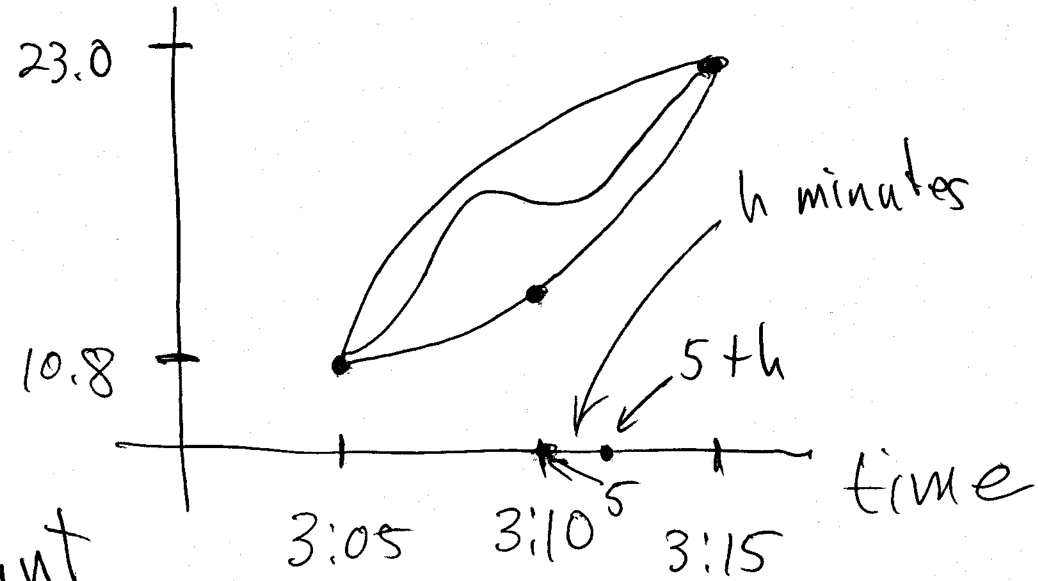
Odometer	Clock
10.8 mi	3:05
23.0 mi	3:15
12.2 mi	10 min $= \frac{1}{6}$ hr.

$$\text{Speed} = \frac{12.2}{\frac{1}{6}} = 73.2 \ \frac{\text{miles}}{\text{hour}}.$$

Average speed over the 10 min period.

Q: How fast was I going exactly at 3:10?

Odometer



Want
Instantaneous speed at <u>3:10</u>

x = time in minutes after <u>3:05</u>

$f(x)$ = odometer reading.

$3:10 \longleftrightarrow x=5$

$$\text{speed} \approx \frac{f(5+h)-f(5)}{h}$$ if h is small.

Inst. speed $\longleftrightarrow$ "$h=0$"

Makes no sense so take $\lim\limits_{h\to 0}$.

2.1 The Derivative

The derivative of a function

The *derivative* of the function $f(x)$ with respect to x is the function $f'(x)$ given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

The process of computing the derivative is called *differentiation*, and we say that $f(x)$ is *differentiable* at $x = c$ if $f'(c)$ exists.

Example

Find the derivative of the function $f(x) = x^2 - 2x$.

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}$$

$$\frac{(x+h)^2-2(x+h)-(x^2-2x)}{h}$$

$$=\frac{\cancel{x^2}+2xh+h^2-\cancel{2x}-2h-\cancel{x^2}+\cancel{2x}}{h}$$

$$=\frac{2xh+h^2-2h}{h}$$

$$=\frac{\cancel{h}(2x+h-2)}{\cancel{h}}=2x+h-2$$

$$(\text{if } h\neq 0)$$

$$=\lim_{h\to 0}(2x+h-2)=2x-2$$

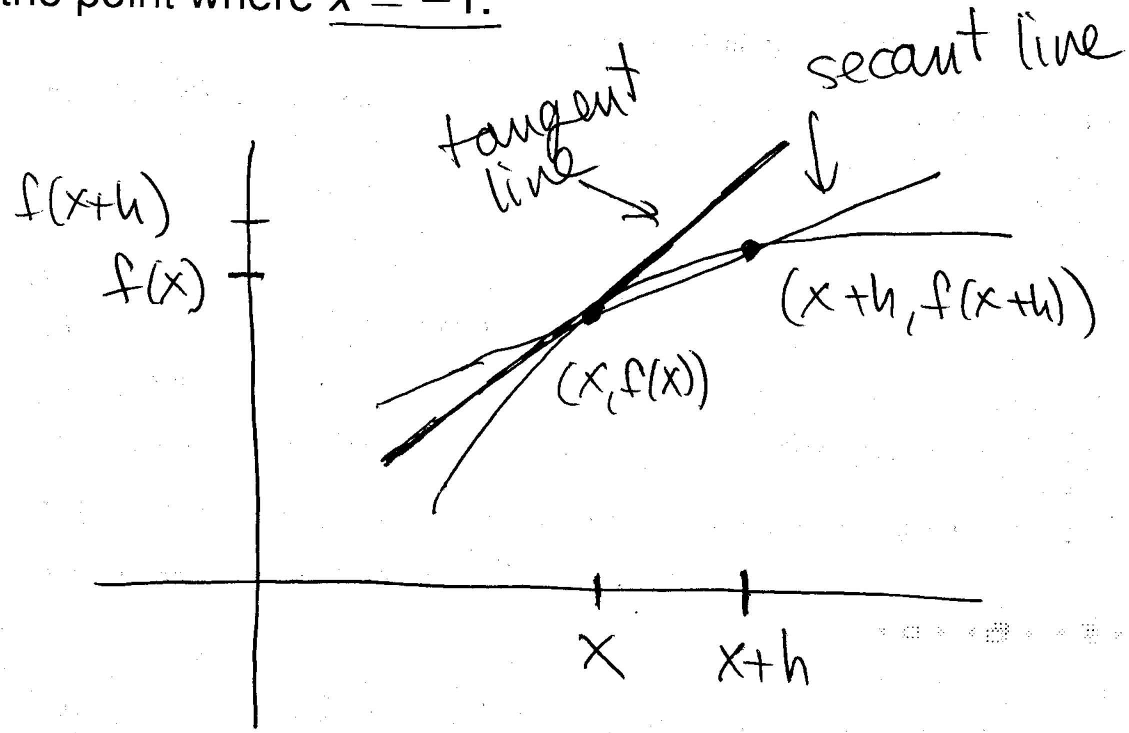
$$f'(x)=2x-2$$

Slope as a Derivative

The slope of the tangent line to the curve $y = f(x)$ at the point $(c, f(c))$ is $m_{tan} = f'(c)$.

Example

Find the equation of the tangent line to the curve $y = x^2 - 2x$ at the point where $x = -1$.



$$\frac{f(x+h)-f(x)}{h} = \text{slope of line through } (x, f(x))\ ((x+h), f(x+h))$$

Equation of line: slope $\leftarrow f'(-1)$

any point

$\uparrow (-1, f(-1))$

$f'(x) = 2x - 2$

$f'(-1) = 2(-1) - 2 = -4$

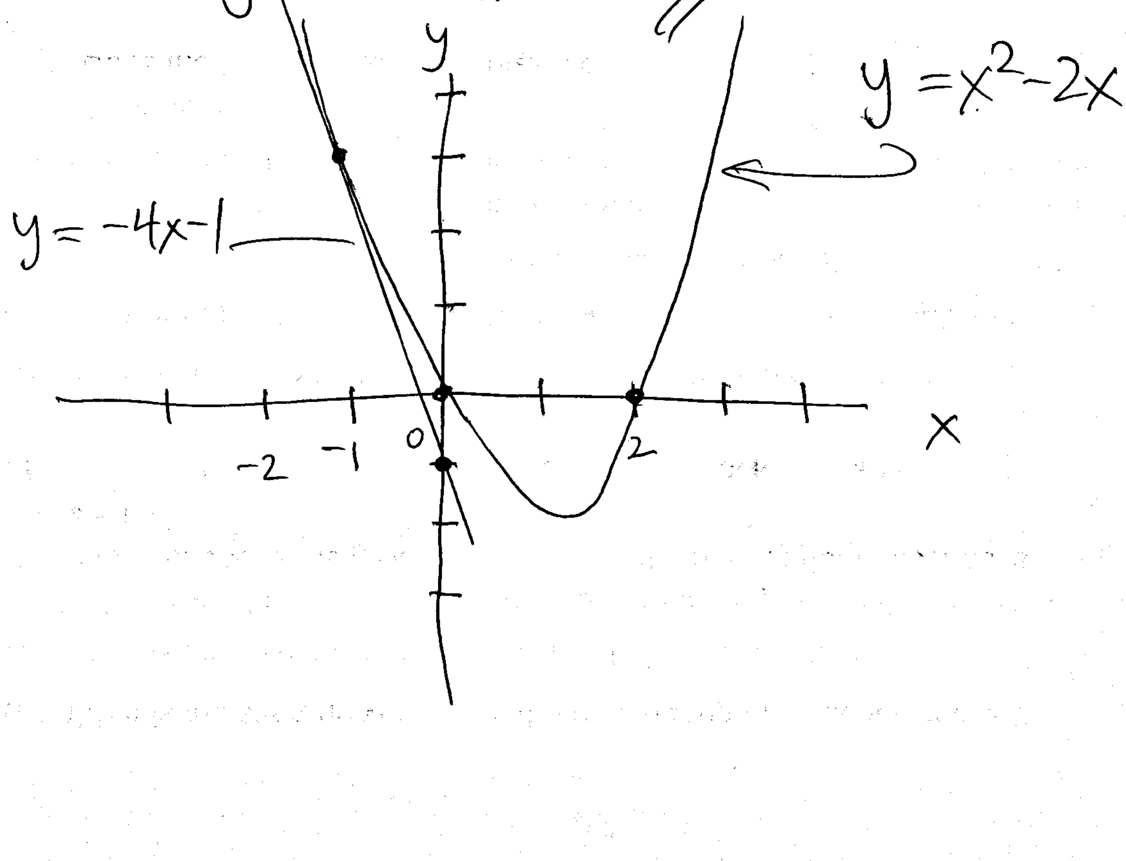
$f(-1) = (-1)^2 - 2(-1) = 1 + 2 = 3$

slope $= -4$ point $(-1, 3)$

$y - 3 = -4(x - (-1))$

$y = 3 - 4(x+1)$ //

$y = -4x - 1$ //



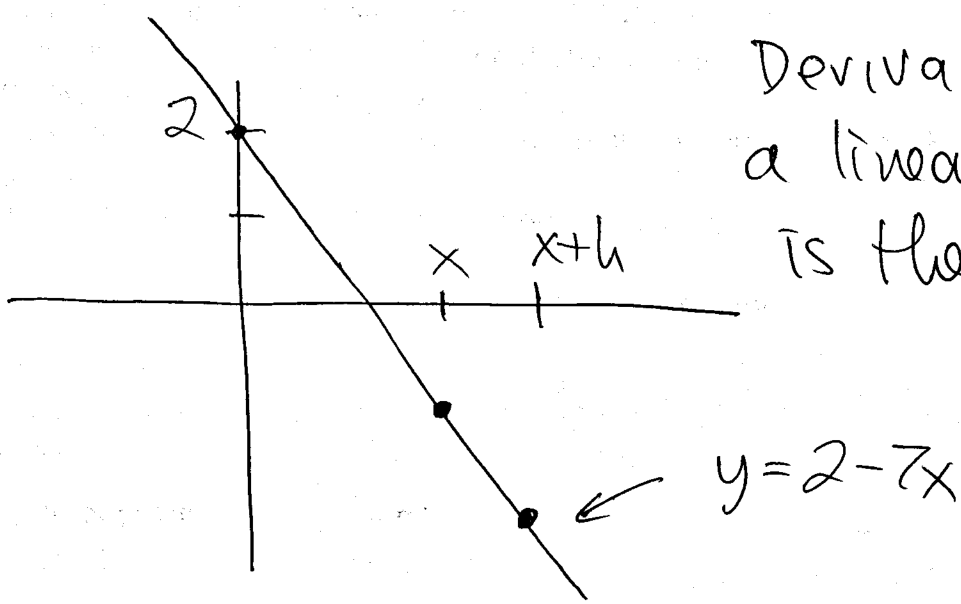
#4 $f(x) = 2 - 7x$ $\quad\quad$ $\underline{x = -1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2 - 7(x+h) - (2 - 7x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2} - \cancel{7x} - 7h - \cancel{2} + \cancel{7x}}{h}$$

$$= \lim_{h \to 0} \frac{-7\cancel{h}}{\cancel{h}} = \lim_{h \to 0} -7 = -7$$



Derivative of a linear function is the $\underline{\text{slope}}$.

#10 $f(x) = \frac{1}{x^2}$ $\underline{x=2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{1}{(x+h)^2} - \frac{1}{x^2}\right)$$

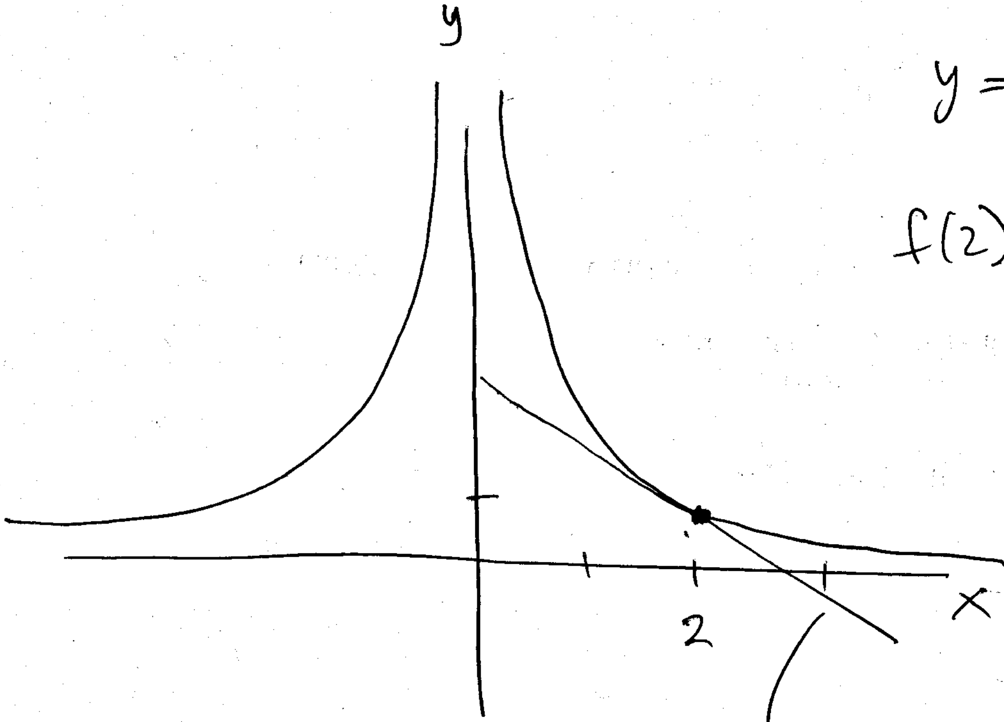
$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{x^2 - (x+h)^2}{x^2(x+h)^2}\right)$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}{x^2(x+h)^2}$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}} \frac{\cancel{h}(-2x - h)}{x^2(x+h)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

Slope of tangent line at $x=2$ is

$$f'(2) = \frac{-2}{(2)^3} = \frac{-2}{8} = -\frac{1}{4}$$



$$y = \frac{1}{x^2}$$

$$f(2) = \frac{1}{4}$$

$$\text{slope} = -\frac{1}{4}$$