MATH 108 – 8 DECEMBER 1998 – EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given.

1. It is estimated that the population in a certain community t years from now will be $P(t) = -t^3 + 9t^2 + 500$ hundred people.

(a) (10 pts.) Find the maximum population over the next 10 years (i.e., for $0 \leq t \leq 10$).

(b) (10 pts.) Find the minimum population over the next 10 years (i.e., for $0 \leq t \leq 10$).

2. (12 pts.) A store will sell 1,000 half–gallon containers of ice cream over the course of a year. Suppose that the cost of storing one half–gallon container of ice cream is 6 dollars per year, and that the ordering cost per shipment is 30 dollars. How large should each shipment be in order to keep total cost at a minimum? (Assume that the ice cream is sold at a constant rate throughout the year and that each shipment arrives just as the previous shipment is used up.)

3. (12 pts.) Tests of a certain artifact reveal that 40% of the original C–14 is still present. Assuming that the amount of C–14 remaining after t years is given by the formula $Q(t) = Q_0\, e^{-.000121t}$, approximately how old is the artifact?

4. It is estimated that the population of prarie dogs in a field t months from now will be $f(t) = \dfrac{6}{1 + 9e^{-.5x}}$ hundred prarie dogs.

(a) (8 pts.) What is the population of prarie dogs 6 months from now?

(b) (8 pts.) At what rate is the population increasing 6 months from now?

(c) (8 pts.) When is the population equal to 300 dogs?

5. Evaluate the derivative of the following functions.

(a) (8 pts.) $f(x) = x^2\, e^{x^2}$

(b) (8 pts.) $\ln\left(\dfrac{x-1}{x+1}\right)$

(c) (8 pts.) $(1 + e^{-4x})^3$

6. (12 pts.) How quickly will money double if it is invested at 8% interest compounded semiannually (twice a year)? How quickly will money double if it is invested at 8% interest compounded continuously? (Hint: The formula for the balance after t years if P dollars is invested at an interest rate of r compounded k times per year is $B(t) = \left(1 + \dfrac{r}{k}\right)^{kt}$. If the interest is compounded continuously, the formula is $B(t) = Pe^{rt}$.)