Math 686: Chapter 3b Homework – Spring 2020

DUE: TUESDAY, FEBRUARY 18, 2020

1. Exercise 3.4 in the textbook.

2. Consider $y' = f(t, y)$ where

$$f(t, y) = -(1 + y^4)y + g(t), \quad y(0) = 1, \quad 0 \le t \le 1,$$

where

$$g(t) = e^{-t}\left[-10\sin(10t) + e^{-4t}(\cos(10t))^5\right].$$

This equation has exact solution $y_{exact}(t) = e^{-t}\cos(10t)$.

Explore numerically using Matlab a suite of 2-state Explicit Runge-Kutta Methods of the form

$$y_{n+1} = y_n + h\left[w_1 f(t_n, y_n) + w_2 f(t_n + \tau_2 h, \xi_2)\right], \qquad n = 1, 2, 3, \ldots,$$

where

$$\xi_2 = y_n + h a_{21} f(t_n, y_n),$$

and $w_2 = 1 - w_1$, $\tau_2 = 1/(2w_2)$ and $a_{21} = \tau_2$ and where w_1 is varied on the interval $[0, 1/2]$.

Make a table of the infinity norm of the error, that is, $\|y - y_{exact}\|_\infty$, for three values $w_1 = 0$, $w_1 = 1/4$ and $w_1 = 1/2$ for $N = 5, 10, 20, 40, 80, 160, 320, 640, 1280$ where the step size $h = 1/N$. You do not need to turn in any plots for this case — just your code and table.

Additionally, for the case $N = 80$ make a plot of the error $\|y - y_{exact}\|_\infty$ as a function of w_1 for $w_1 \in [0, 1/2]$ with enough resolution (i.e. enough values of w_1) for a smooth curve. What is your estimate for the best value of w_1 to minimize the error? Test out another ODE with a different exact solution to test the hypothesis that the best w_1 is problem dependent. You don't need to report a detailed error analysis here but make a comment about the new optimal w_1.