Math 113: Quiz 9

Instructions: Answer all questions. Show all of your work. Partial credit may be given. The use of calculators is not allowed. Please turn off and put away all mobile electronic devices - accessing these devices between the time you receive your quiz and the time you turn in your quiz constitutes an honor code violation.

1. (6 pts) Use L’Hôpital’s rule to evaluate the following limits

   (a) \( \lim_{x \to 1} \frac{x^3 - x^2 - x + 1}{x - 1} = \) \( \lim_{x \to 1} \frac{3x^2 - 2x - 1}{1} \) = \( 0 \)

   Indeterminate Form

   Type \( 0 \over 0 \)

   (b) \( \lim_{x \to 0} \frac{x^2}{e^{4x} - 1} = \) \( \lim_{x \to 0} \frac{2x}{4e^{4x}} \) = \( 0 \)

   Indeterminate Form

   Type \( 0 \over 0 \)

   (c) \( \lim_{x \to \infty} \frac{x^2}{10^x} = \) \( \lim_{x \to \infty} \frac{2x}{10^x \ln 10} \) \( \lim_{x \to \infty} \frac{2}{10^x (\ln 10)^2} = \) \( 0 \)

   Indeterminate Form

   Type \( \infty \over \infty \)

2. (4 pts) Consider the fenced in region adjacent to a building where the exterior fencing parallel to the building costs $4 per unit length and two sides perpendicular to the building costs $6 per unit length (see diagram).

   (a) Write down a function \( C(\ell) \) that represents the cost of the fence as a function of the length \( \ell \) if the total fenced in region is required to have area \( A_0 \).

   \[
   \text{Cost} = 4W + 6\ell + 6\ell = 4W + 12\ell
   \]

   \[
   A_0 = \ell W \quad \text{so} \quad W = \frac{A_0}{\ell}
   \]

   \[
   C(\ell) = \frac{4A_0}{\ell} + 12\ell
   \]

   (b) Identify the optimal value of \( \ell \) such that the cost of the fence is minimized.

   \[
   C'(\ell) = -\frac{4A_0}{\ell^2} + 12 \leq 0 \quad \text{when} \quad \ell^2 = \frac{4A_0}{12} = \frac{A_0}{3}
   \]

   First Deriv. Test

   \[
   C_{\ell_0} = C'_{\ell_0} = 0
   \]

   \[
   C_{\ell_0} \text{ has min when } \ell = \ell_c
   \]

   \[
   \ell_c = \sqrt[3]{\frac{A_0}{3}}
   \]