Math 113: Quiz 3

Instructions: Answer all questions. Show all of your work. Partial credit may be given. The use of calculators is not allowed. Please turn off and put away all mobile electronic devices - accessing these devices between the time you receive your quiz and the time you turn in your quiz constitutes an honor code violation.

1. (3 pts) Which one-sided limits should be examined if you want to determine if the function \( f(x) \) listed below has a vertical asymptote at \( x = 0 \)? (you do not need to evaluate these limits)

\[
f(x) = \frac{x + 2}{x^4}
\]

\[
\lim_{x \to 0^+} \frac{x + 2}{x^4} \quad \quad \quad \quad \lim_{x \to 0^-} \frac{x + 2}{x^4}
\]

2. (4 pts) Identify, by evaluating the appropriate limit(s), all vertical asymptotes and all horizontal asymptotes of the function.

\[
f(x) = \frac{x}{x + 6}
\]

\[
\begin{align*}
\lim_{x \to \infty} \frac{x}{x + 6} &= \lim_{x \to \infty} \frac{1}{1 + \frac{6}{x}} = 1 \\
\therefore f \text{ has horizontal asymptote } y = 1 \text{ as } x \to \infty
\end{align*}
\]

\[
\begin{align*}
\lim_{x \to -\infty} \frac{x}{x + 6} &= \lim_{x \to -\infty} \frac{1}{1 + \frac{6}{x}} = 1 \\
\therefore f \text{ has horizontal asymptote } y = -1 \text{ as } x \to -\infty
\end{align*}
\]

\[
\begin{align*}
\lim_{x \to -6^+} \frac{x}{x + 6} &= -\infty \\
\lim_{x \to -6^-} \frac{x}{x + 6} &= +\infty
\end{align*}
\]

\[
\therefore f \text{ has vertical asymptote } x = -6
\]

3. (3 pts) Use the limit definition of derivative to show that the derivative of the function \( f(x) = x^2 + 5x \) is \( f'(x) = 2x + 5 \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + 5(x+h) - (x^2 + 5x)}{h}
\]

\[
= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 5x + 5h - x^2 - 5x}{h} = \lim_{h \to 0} \frac{2xh + h^2 + 5h}{h} = \lim_{h \to 0} 2x + h + 5
\]

\[
= 2x + 5
\]