Math 113: MATHEMATICA Assignment 1

DUE: THURSDAY, OCTOBER 10, 4:30PM

Instructions: You may work on this in groups of up to three people. If you work in a group please hand in *only one* copy with all of the group members' names on it. An *environmental fee* will be charged to groups that do not follow this rule. NOTE: My definition of a group includes equal contributions from all members on all problems. All group members should be able to explain any result turned in for credit on this assignment. If your group falls apart mid-way through the assignment please state clearly who did which problem. ALSO NOTE: With probability very close to one, no two groups' assignments should look alike. If four of you work together and print off two versions of the same assignment pretending to be two groups I will know and will reduce scores by an appropriate factor (2 in this example).

IMPORTANT: Please answer questions in a neat and organized manner. Specifically, this means that you should use complete sentences and make sure that any graph you expect me to understand (and grade!) has axes and curves labelled (you can do this by hand after you print the graph if that is easiest). Note that different colored curves look the same when printed in black and white. Also, recognize that quantity of pages, graphs and numerical output does not necessarily correspond to quality. Please do not turn in Mathematica output without accompanying explanations. Feel free to suppress failed attempts when printing pages.

You have over two weeks to complete this assignment but completing it will require that you get to a computer lab on campus or install and run the mathematica software on your own computer. Plan to finish it early. There will be 10% deduction in your score if you do not turn in a hard copy of this assignment at the beginning of class on the due date or earlier. Every subsequent calendar day (counting weekend days and spring break days) late will result in an additional deduction (two days = 20%, three days = 30%, etc.) until I have completed grading the assignments at which time no further late assignments will be accepted.

Pro Tip 1: *There are helpful commands listed at the end of this assignment. Read through them and make sure you can successfully run them before starting on the assignment.*

Pro Tip 2: *Complete Pro Tip 1 now.*

Pro Tip 3: *There are multiple formats in which you can enter commands in Mathematica. I recommend (and describe in these notes) the 'Wolfram Languange input [default]'. If you choose to use another format such as 'Free-form input' or 'Wolfram : Alpha query' the recommendations and examples I've listed may not be very helpful.*

1. (*15 pts*) Evaluate the limit (or determine that it DNE)

$$\lim_{x \to 0} f(x)$$

where

$$f(x) = \frac{\sin(x^2) - \sin^2 x}{x^4}$$

in three ways as described in parts (a), (b), and (c).

(a) Calculate and write out a table of function values for the function $f(x)$ as x approaches 0 (use Mathematica to calculate the function values but you can write out the table by hand on a separate piece of paper if it is easier). The whole point of making the table is to help evaluate this limit (what value does the function approach?) so choose your values of x wisely. Write a sentence to give a conclusion about what you think the value of this limit is based on the information you have compiled in your table.

Pro Tip: *Note that* $\sin(x)$ *in Mathematica is* `Sin[x]`, $\sin(x^2)$ *is* `Sin[x^2]` *and* $\sin^2 x$ *is* `Sin[x]^2`

Pro Tip: *In order to confirm that you have your function defined correctly in the Mathematica software it might be helpful to compute the value of this function at few values of* x *using some other method or device (e.g. your favorite calculator). If you cannot get Mathematica to evaluate your function correctly you should focus your attention on correcting this problem – which in most cases means that the function is not defined properly – rather than proceeding to part (b).*

(b) Graph the function for values of x close enough to 0 so that you can determine the value of this limit to within 2 decimal places of accuracy simply by looking at the graph. Write a sentence or two describing your conclusions about the limit from the graph and compare with the results you found in part (a).

Pro Tip: *Remember here [and in part (a)] that the question is asking about the limit of this function as* x *approaches* 0 *so a graph showing the function on an interval* $[-100, 100]$ *might look nice or interesting but probably will not help answer the question.*

(c) Use Mathematica's limit command. Write one or two sentences summarizing the results in parts (a), (b), and (c). Are the results consistent in predicting the value of the limit?

Pro Tip: *In all of these cases keep in mind what you might already know about the function. If you see a result that you don't understand, question it and try to find out what might be wrong. Computers make numerical approximations that can involve errors. Programmers (you) make (sometimes subtle) mistakes but hopefully your team can catch these mistakes. The software does what you tell it to do, not necessarily what you expect or want it to do.*

2. (*15 pts*) Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{\sin(x^2) - \sin^2 x}{x^4}$$

Your answers should provide justification and explanation. The supporting evidence for your answer could include graphical results, results of limits, connections to definitions of limits that we have introduced in class, etc.

Pro Tip: *Go back and review what is required for a function to have an asymptote. Try to apply those ideas to this function.*

3. (*20 pts*) The limit definition of the derivative of a function is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

One way to obtain an approximation of the derivative $f'(x)$ is to approximate the limit by evaluating the expression

$$\frac{f(x+h) - f(x)}{h},$$

for a small but nonzero value of h. With this in mind, consider the following expression that can be thought of as a function depending on two quantities x and h

$$DQ(x,h) = \frac{\sin(x+h) - \sin x}{h}$$

Plot $DQ(x,h)$ on the x-interval $[-2\pi, 2\pi]$ for different fixed values of $h = 2$, 1, 0.5, 0.25, 0.125, 0.0625 (put these all on the same graph). Can you identify a function of x that resembles $DQ(x,h)$ when h is small?

Pro Tip: *Note that* 2π *is* `2*Pi` *in Mathematica.*

Pro Tip: *See hints in the back. The plot command* `Plot[{DQ[x,0.5], DQ[x,0.25]}, {x, xmin, xmax}]` *will put two graphs on the plot.*

Pro Tip: *Evaluate the limit* $\lim_{h \to 0} DQ(x,h)$ *to get a good clue about the function.*

Some helpful Mathematica commands:

- π is `Pi`, e^x is `Exp[x]`, $\ln x$ is `Log[x]`, $\log_b x$ is `Log[b,x]`, $\sin x$ is `Sin[x]`, $\cos x$ is `Cos[x]`, $\tan x$ is `Tan[x]`, $|x|$ is `Abs[x]`, $\tan^{-1} x$ is `ArcTan[x]`. For example, note that if you enter `Sin x , sin x , sin(x), sinx, Sinx, Sin(x)` instead of `Sin[x]` don't expect Mathematica to know what you are talking about.

- Define a function $g(x) = x^2 + 4x + 3$:

```
g[x_] := x^2 + 4*x + 3
```

Note the underscore ('blank') after the x.

- Evaluate $g(0.01)$ as a floating point number with 12 decimal places listed:

```
NumberForm[g[0.01], 12]
```

- To define a function of more than one variable (e.g. $G_1(x, h) = x + h$):

```
G1[x_,h_] := x + h
```

- Display a graph (for a properly defined function):

```
Plot[g[x], {x, xmin, xmax}, PlotRange ->{ymin, ymax}]
OR
Plot[g[x], {x, xmin, xmax}]
OR
Plot[G1[x,0.5], {x, xmin, xmax}]
```

where you must supply numerical values of xmin, xmax, etc. The y-range is optional. Note, once you've got the plot command working, it is very easy to change the min and max values of x and y and to zoom in or out. **If your plot is blank and you think it should not be blank there is a good chance that your function is not properly defined.**

- Display multiple graphs in the same figure:

```
Plot[{f[x], g[x], h[x]}, {x, -2, 2}]
```

where you must have already defined f, g and h.

- The limit command $\lim_{x\to a} g(x)$:

```
Limit[g[x], x -> a]
```

where you must supply the value of a and have already defined the function g.

To specify a right-sided limit, $\lim_{x\to a^+} g(x)$

```
Limit[g[x], x -> a, Direction -> 1]
```

To specify a left-sided limit, $\lim_{x\to a^-} g(x)$

```
Limit[g[x], x -> a, Direction -> -1]
```