

Weekly Homework 4

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Topos Theory

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Problem 1. The Inverse Image Functor

Let $f : X \rightarrow Y$ be a continuous map of topological spaces. Show that the direct image functor

$$f_* : \mathbf{Sh}(X) \rightarrow \mathbf{Sh}(Y)$$

has a left exact left adjoint f^* , i.e. a left adjoint which preserves finite limits. The functor f^* is called the **inverse image functor**.

Hint: Use Problem 2d) from Homework 3 (Note: Homework 3 has been updated).

Problem 2. The Associated Sheaf Functor

Denote the left adjoint to the inclusion

$$i : \mathbf{Sh}(X) \hookrightarrow \mathbf{Set}^{\mathcal{O}(X)^{op}}$$

by a . Prove that a is left exact.