# Weekly Homework 3

Instructor: David Carchedi
Topos Theory

May 8, 2013

## Problem 1. An exercise on stalks

(a) Let $F$ be a presheaf on $X$. Consider for each open $U$ the canonical map

$$F\left(U\right) \xrightarrow{\prod\limits_{x\in U} germ_x(U)} \prod_{x\in U} F_x.$$

Show that this map is injective if and only if $F$ is separated.

(b) Show that if $\mu$ and $\lambda$ are elements of $F\left(U\right)$, with $F$ any presheaf, then the set

$$\{x \in U \,|\, germ_x\mu = germ_x\lambda\}$$

is open in $U$.

## Problem 2. Local Homeomorphisms

**Definition 1.** A map $f : E \to X$ between topological spaces is said to be a local homeomorphism if for each point $e \in E$, there exists a neighborhood $U$ of $e$ in $E$ such that $f$ restricts to a homeomorphism of $U$ onto a neighborhood of $f(e)$.

(a) Prove that a continuous map $f : E \to X$ is a local homeomorphism if and only if $f$ is open and the diagonal of $f$ is open, where the diagonal of $f$ is the canonically induced map

$$\Delta_f : E \to E \times_X E.$$

(b) Show that a local homeomorphism is a homeomorphism if and only if it is bijective.

(c) Show that if two sections of a local homeomorphism agree at a point, then they agree on a neighborhood of that point.

(d) Show that if $f : E \to X$ is a local homeomorphism, and $p : Y \to X$ is any continuous map, that the induced map

$$Y \times_X E \to Y$$

is a local homeomorphism.

(e) Prove that if

$$F \xrightarrow{f} E \xrightarrow{g} X$$

are continuous maps, then

i) If $f$ and $g$ are local homeomorphisms, then so is $gf$

ii) If $g$ and $gf$ are local homeomorphisms, then so is $f$.

iii) If $f$ and $gf$ are local homeomorphisms, then so is $g$ *if* $f$ is surjective. Give a counterexample otherwise.