TYRUS BERRY

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RESEARCH POSITIONS

Sept 2015-Aug 2017 *Post Doctoral Research Fellow*, Department of Mathematical Sciences, George Mason University

Aug 2013-July 2015 *Research Associate*, Department of Mathematics, Pennsylvania State University

EDUCATION

2013 **PhD in Mathematics** from George Mason University. GPA: 4.0

- **Thesis:** *Model-Free Techniques for High Dimensional Dynamics*
- **Advisor:** Timothy Sauer

2008 **MS in Mathematics** from Ohio State University. GPA: 3.5

- **Thesis:** *An Overview of Optimal Stopping Times for Various Discrete Time Games*

2006 **BS in Applied Mathematics** and
BA in Physics from the University of Virginia. GPA: 3.9

- Highest Distinction, School of Engineering and Applied Sciences

RESEARCH INTERESTS

Geometry of data: diffusion maps/local kernels, nonlinear dimensionality reduction and decomposition.
Statistics: kernel density/operator estimation, semiparametric modeling of dynamical systems.
Dynamical systems: data assimilation, prediction/control, and uncertainty quantification.
Harmonic analysis: Sampling theory on manifolds, connections to kernel based statistical estimates

PUBLICATIONS

Articles in Refereed Journals

1. T. Berry, J. Harlim, *Iterated Diffusion Maps for Feature Identification.* In Press, Journal of Applied and Computational Harmonic Analysis, 2016.
`http://dx.doi.org/10.1016/j.acha.2016.08.005`

   Diffusion maps were designed to find low-dimensional coordinates for high-dimensional data. In the theory of *local kernels* (Berry and Sauer, ACHA 2015), the theory underlying diffusion maps was extended to represent an unknown diffeomorphism between two data sets. In this paper we extend this theory to arbitrary feature maps, where the feature may be intrinsically lower dimensional than the data set. Given a training data set with known feature values, the iterated diffusion map (IDM) approximates a geometric flow which represents the feature map. Key theoretical results include the rigorous estimation of the derivative of an unknown mapping between unknown manifolds, and rigorous identification of an unknown tangent space via local scaling laws.

2. T. Berry, T. Sauer, *Density estimation on manifolds with boundary.* Computational Statistics and Data Analysis, Volume 107, pp. 1-17, 2017 (to appear). `http://dx.doi.org/10.1016/j.csda.2016.09.011`

   In this paper we introduce a kernel density estimation (KDE) method for data lying on an embedded manifold, where the geometric structure of the data is completely unknown. For the purposed of density estimation, we extend the theory of local kernels (Berry and Sauer, ACHA 2015) to the large class of *tangible* manifolds, which includes all compact manifolds and a large class of non-compact manifolds. The theory recovers classical KDE on Euclidean manifolds as a particular example. We then generalize the cut-and-normalize boundary correction to embedded Riemannian manifolds with unknown geometry and unknown boundary. To achieve this generalization, we introduce rigorous estimators for the distance and direction of the boundary.

3. T. Berry, J. Harlim, *Semiparametric forecasting and filtering: correcting low-dimensional model error in parametric models.* Journal of Computational Physics, Volume 308, pp. 305-321, 2016. `http://www.sciencedirect.com/science/article/pii/S0021999115008621`

   This paper generalizes the statistical semiparametric framework to dynamical systems modeling. Given a physically motivated parametric model subject to model error, and a training data set of noisy physical observations, our approach learns an auxiliary nonparametric model which 'repairs' the model error. The nonparametric model is the diffusion forecast method introduced in Berry, Giannakis, and Harlim (PRE 2015) and algorithms are developed for semiparametric forecasting and filtering.

4. F. Hamilton, T. Berry, T. Sauer, *Ensemble Kalman filtering without a model.* Phys. Rev. X 6, 011021, 2016. `http://dx.doi.org/10.1103/PhysRevX.6.011021`

   We merge Takens' method for state space reconstruction with nonlinear Kalman-based filtering to remove observation noise from dynamical data without assuming any model. This procedure replaces the model with dynamics reconstructed from delay coordinates, while using the Kalman update formulation to reconcile new observations. We find that this combination of approaches results in comparable efficiency to parametric methods in identifying underlying dynamics, and may actually be superior in cases of model error.

5. T. Berry, J. Harlim, *Forecasting Turbulent Modes with Nonparametric Diffusion Models.* Physica D, Volume 320, pp. 57-76, 2016. `http://dx.doi.org/10.1016/j.physd.2016.01.012`

   This paper shows how the diffusion forecast algorithm introduced in Berry, Giannakis and Harlim (PRE 2015) can be used to find a reduced model for turbulent modes arising from complex spatiotemporal dynamics. A nonparametric Bayesian filter that estimates the initial state for the diffusion forecast from noisy observations is introduced.

6. T. Berry, D. Giannakis, J. Harlim, *Nonparametric forecasting of low-dimensional dynamical systems.* Phys. Rev. E 91, 032915, 2015. `http://dx.doi.org/10.1103/PhysRevE.91.032915`

   For a time series generated by a stochastic dynamical system on a manifold, we connect the discrete shift map with the semigroup solution of the dynamical system. By projecting the shift map onto a basis of smooth functions adapted to the geometry and the invariant measure, we obtain an optimal discrete approximation to the solution semigroup from a training data set. The resulting algorithm can forecast a full density giving improved forecasting compared to locally linear forecasting. The method is verified on a stochastic evolution on a torus embedded in Euclidean space and then applied to the deterministic and chaotic Lorenz-63 system, as well as a real data set representing the El Nino index.

7. T. Berry, T. Sauer, *Local Kernels and the Geometric Structure of Data.* Journal of Applied and Computational Harmonic Analysis, Volume 40, Issue 3, pp. 439-469, 2015. `http://dx.doi.org/10.1016/j.acha.2015.03.002`

   The theory underlying diffusion maps is extended to a large class of kernels called *local kernels*. The diffusion maps theory developed by Coifman and Lafon originally showed that a isotropic kernel on a data set could be used to represent the geometry inherited from the ambient space, and in particular could yield discrete approximations to the Laplace-Beltrami operator. Here, we show that the anisotropy in a local kernel, combined with the embedding geometry of a data set, determines the resulting geometry represented by the kernel. Moreover, for any desired Riemannian geometry on a data set there exists a local kernel that represents the desired geometry.

8. T. Berry, J. Harlim, *Variable Bandwidth Diffusion Kernels.* Journal of Applied and Computational Harmonic Analysis, Volume 40, Issue 1, pp. 68-96, 2015. `http://dx.doi.org/10.1016/j.acha.2015.01.001`

   The theory underlying diffusion maps is extended to variable bandwidth kernels. In analogy to the variable bandwidth kernel density estimation often applied in statistics to improve estimation of the tails of a density, it is found that variable bandwidth kernels can be used to extend the diffusion maps results to non-compact manifolds for the first time.

9. T. Berry, J. Harlim, *Nonparametric Uncertainty Quantification for Stochastic Gradient Flows.* SIAM/ASA Journal on Uncertainty Quantification, 2015, Vol. 3, No. 1, pp. 484-508. `http://dx.doi.org/10.1137/14097940X`

   We present a novel uncertainty quantification approach for stochastically forced gradient flow systems on manifolds based on using the diffusion maps algorithm with a variable bandwidth diffusion kernel. Our approach learns the model from a training data set, assuming that the true evolution is described by an unknown stochastic gradient flow on an unknown manifold and the approach is nonparametric in the sense that no parametric form is assumed for either the manifold or the gradient flow system. We show how to use the approximate semigroup solution to solve the fully nonlinear forecasting, filtering and response problems.

10. F. Hamilton, T. Berry, T. Sauer, *Predicting chaotic time series with a partial model.* Phys. Rev. E 92, 010902 2015. `http://dx.doi.org/10.1103/PhysRevE.92.010902`

    We present a novel approach for combining partial information about the model governing a dynamical system with time-delay embedding based forecasting. Using the partial equation, we are able to interpolate the data and the larger data set provides better surrogates for out-of-sample forecasting.

11. T. Berry, J. Harlim *Linear theory for filtering nonlinear multi scale systems with model error.* Proceedings of the Royal Society A 470, 2014. `http://dx.doi.org/10.1098/rspa.2014.0168`

    We consider the problem of filtering with an imperfect model due to unresolved fast time scale variables. In the linear case we improve on the results of averaging theory and show that a consistency condition yields an optimal choice of reduced model that matches the mean and covariance estimates of the full model. By considering a simple nonlinear multi-scale problem with additive Brownian forcing, we find that a multiplicative noise term is needed to compensate for the unresolved scales. Finally, we consider a general ansatz for compensating for unresolved scales, and show dramatic improvement in a two-layer Lorenz96 model with 81 variables, 72 of which are unresolved in the reduced model.

12. T. Berry, R. Cressman, Z. Greguric-Ferencek, T. Sauer, *Time-scale separation from diffusion-mapped delay coordinates.* SIAM J. Appl. Dyn. Sys. 12, pp. 618-649, 2013.
`http://dx.doi.org/10.1137/12088183X` Videos: `http://math.gmu.edu/~tsauer/dmdc.html`

    We extend the classical state space reconstruction of Takens in order to find the latent geometry of a dynamical system from an observed time series using a model-free, data driven approach. We give a new interpretation to the diffusion maps algorithm of Coifman and Lafon for time series, and show that this is a natural approach for achieving time-scale separation. We develop our approach into an algorithm called Diffusion Mapped Delay Coordinates (DMDC).

13. T. Berry, T. Sauer, *Adaptive ensemble Kalman filtering of nonlinear systems.* Tellus A 65, 20331, 2013. `http://dx.doi.org/10.3402/tellusa.v65i0.20331`

    We develop a novel technique for adaptively determining the covariance matrices of system and observation noise using the innovation errors of an ensemble Kalman filter. We show that our technique leads to significantly improved state estimates, and when the noise is non-autonomous our method can automatically track the changing covariance structure. Finally, an adaptive filter can compensate for model error by automatically inflating the system noise covariance which reduced estimation error from 200% higher than the level of no model error to just 20%.

14. F. Hamilton, T. Berry, N. Peixoto, and T. Sauer, *Real-time tracking of neuronal network structure using data assimilation.* Phys. Rev. E 88, 052715, 2013.
`http://dx.doi.org/10.1103/PhysRevE.88.052715`

    Building on the success of the Cox method statistical test for detecting changes in connections, this paper addresses the large data requirements of the Cox method. We use the Hindmarsh-Rose model for the neurons at the nodes of the network, and connect these nodes via the variables representing the intracellular potential. We use an ensemble Kalman filter to estimate the parameters of the individual neurons as well as parameters which quantify the connection strengths.

15. T. Berry, F. Hamilton, T. Sauer, N. Peixoto, *Detecting connectivity changes in neuronal networks.* Journal of Neuroscience Methods 209, pp. 388-397, 2012.
`http://dx.doi.org/10.1016/j.jneumeth.2012.06.021`

    We refine previous work on the Cox method for detecting connections in neuronal networks and improve the multiple hypothesis testing technique. We develop a new statistical test to allow experimental scientists to use the Cox method to determine if a connection strength has changed. We show that the technique is independent of the neuron model and robust to choice of model for the connections and apply the technique to experimental data.

16. T. Berry, T. Sauer, *Convergence of periodically-forced rank-type equations.* Journal of Difference Equations and Applications 17, 2011. `http://personal.psu.edu/thb11/forcedranktype.pdf`

    Previous results on the convergence of rank-type difference equations are subsumed in a unified theory of sup-contractive difference equations. We show that sup-contractive equations have a limiting periodic behavior with period no more than the forcing period, and independent of the 'memory' of the difference equation.

17. T. Berry, S. Heilman, R. Strichartz, *Outer approximation of the spectrum of a fractal laplacian.* Experimental Mathematics 18 no. 4 pp. 449-480, 2009.
`http://personal.psu.edu/thb11/OuterApproximationV6.pdf`

Articles Submitted for Publication

- T. Berry, T. Sauer, *Consistent Manifold Representation for Topological Data Analysis*. Submitted to Foundations of Computational Mathematics, 2016.
`http://math.gmu.edu/~berry/Publications/TDA.pdf`

  We introduce the Continuous k-Nearest Neighbors (CkNN) graph construction and prove that it is the unique construction that is consistent with the underlying manifold topology. The theory applies to compact Riemannian manifolds and a large class of non-compact manifolds. In contrast to persistent homology, which represents each homology generator at a separate scale, CkNN produces a single graph that captures all topological features simultaneously. This requires new results on the spectral convergence of the graph Laplacian as a consistent estimator of the Laplace-de Rham operator. As applications, we derive a new fast clustering algorithm and a method to identify patterns in natural images topologically.

- J. Harlim, T. Berry, *Correcting biased observation model error in data assimilation.* Submitted to Monthly Weather Review, 2016. `http://math.gmu.edu/~berry/Publications/RKHS.pdf`

  We generalize a data-driven method of estimating conditional probability density functions to apply to weighted reproducing kernel Hilbert spaces (RKHS). We then apply this method correct observation model error in filtering problems using historical training data. We develop a framework for learning the conditional density of the observation model error, given the current observation, and use a Bayesian update to correct the bias in each the observation. This framework is a 'secondary filter' which can be incorporated into any existing filtering method. We focus on the application of assimilating satellite observations in the presence of clouds using existing models for cloud formation and radiative transfer models. When cloud formation is unpredictable, or effects the observation function in unknown ways, existing model can lead to significant errors, often requiring cloudy observations to be thrown out. Our secondary filter corrects these observations and achieves significantly improved estimates of the state variables.

- F. Hamilton, T. Berry, T. Sauer, *Kalman-Takens filtering in the presence of dynamical noise.* Submitted to The European Physical Journal ST, 2016.
`http://math.gmu.edu/~berry/Publications/KTnoisy.pdf`

  We generalize the Kalman-Takens model-free filtering method to stochastic systems. We address the significant challenge of separating dynamical and observational noise without using a parametric model. By combining Takens' state space reconstruction theory to stochastic data (using a generalization of Broomhead and Stark) we are able to estimate the local dynamics. The adaptive filtering method we previously introduced in (Berry and Sauer, 2013) is shown to correctly separate the observation and system noise covariances for a stochastically forced Lorenz-63 model.

TEACHING EXPERIENCE

- *Instructor*, George Mason University, Fall 2016.
  - *Math 685 / Operations Research 682, Numerical Methods:* Changed the course book to *Numerical Mathematics* by A. Quarteroni et al. with the goal of introducing more mathematical rigor while still serving out-of-department graduate students. See Fall 2015 below and `http://math.gmu.edu/~berry/685Fall2016/index.html` for more information.

- *Chair, Numerical Analysis Preliminary Exam Committee*, George Mason University, Fall 2016.
  - Coordinated the committee in charge of writing, administering, and grading the preliminary examination for Numerical Analysis based on the Math 685 coursework.

- *Co-Instructor*, George Mason University, Fall 2016.
  - *Reading Course in Riemannian Geometry:* This course was designed for Ryan Vaughn, a graduate student interested in topological and geometric methods in data science. Based on *Introduction to Smooth Manifolds* and *Riemannian Manifolds*, both by John M. Lee. The goal has been to develop a solid foundation in the fundamentals of Riemanian geometry while emphasizing aspects that are critical to data science, such as embedding theorems, connections between intrinsic and extrinsic distances, and properties of the exponential map.
- *Instructor*, George Mason University, Fall 2015.
  - *Math 685 / Operations Research 682, Numerical Methods:* This course prepares mathematics graduate students to take the preliminary exam in Numerical Analysis. It covers a broad range of topics including floating point arithmetic, numerical linear algebra, eigen-solvers, nonlinear solvers/optimization, approximation theory, numerical integration/differentiation, and ODE/PDE solvers. These topics are unified by the central concepts of conditioning, stability/convergence, and algorithmic efficiency. The course is also attended by many out-of-department graduate students whose research requires intensive numerical and scientific programming.
- *Organizer, Nonlinear Data Analysis Seminar*, George Mason University, Fall 2015.
  - Organized and made weekly presentations on manifold learning methods in data science, including foundations, current research, and future directions. The seminar was attended weekly by 6-10 graduate students and 1-2 undergraduate students, as well as 3-4 faculty members. As of Fall 2016 the seminar is currently transitioning into a graduate student led seminar with an emphasis on deep learning methods. See `http://math.gmu.edu/nda-seminar.php` for slides and more information.
- *Instructor*, George Mason University, Summer Session 2013.
  - *Math 446 / Operations Research 481, Numerical Methods:* This course introduces the fundamental concepts of conditioning of problems and algorithmic efficiency and covers numerical methods for floating point arithmetic, numerical linear algebra, nonlinear solvers, interpolation, numerical integration/differentiation and ODE solvers. Programming assignments are based on MATLAB. This is a medium-sized lecture course (40-60 students) targeted mostly to math and computer science majors but also attended by other science and engineering majors who use extensive numerical/scientific programming.
- *Instructor*, Ohio State University, Summer Session 2007.
  - Pre-Calculus
- *Teaching Assistant*, Ohio State University, 2006-2008.
  - Pre-Calculus
  - Calculus I
  - Calculus II

MENTORING UNDERGRADUATE AND GRADUATE RESEARCH

- *Assistant Mentor, Graduate Student Research*, George Mason University, 2015-2016
  Assisted my advisor, Timothy Sauer, in mentoring three graduate students, Franz Hamilton, Marilyn Vazquez and Ryan Vaughn. Member of Marilyn Vazquez's thesis committee. Participated in weekly meetings and helped guide their research projects.

  - Franz Hamilton graduated in May 2015 and is currently a postdoctoral research fellow at NCSU. Franz was a cross-disciplinary graduate student who worked on neural engineering in the Volgenau School of Engineering at GMU and also had close ties to the Mathematical Sciences department. Tim Sauer and I worked together extensively with Franz starting in 2013 to learn the fundamentals of Kalman filtering and then to develop Kalman filtering based methods of learning and tracking features of neural networks from noisy data. We have co-authored several papers and continue to collaborate and develop new filtering techniques for addressing model error.
  - Marilyn Vazquez is developing a new method of unsupervised image segmentation based on clustering and classification of sub-images. We have worked together extensively; reading the clustering literature and implementing/testing existing and new algorithms, as she develops her new approach based on density estimation and persistent homology.
  - Ryan Vaughn is a graduate student with a background in topology who is interested in topological data analysis (TDA). Over the summer of 2016 we explored the extensive literature in this field together, and developed a MATLAB implementation of key TDA algorithms (previously only available on other platforms). We have organized a reading course for him in Riemannian geometry with a focus on topics required in our data science research.

- *Seminar Instructor, Introduction to Data Science for EXTREEMS Undergraduate Research Program*, George Mason University, Summer 2016.
  - Organized a three day seminar introducing the new undergraduate research students to fundamental concepts and methods in data science, including principal component analysis (PCA), multi-dimensional scaling (MDS), nonlinear dimensionality reduction, density estimation, and nonparametric modeling/forecasting. Each day consisted of a lecture followed by a supervised data analysis and programming project and a small homework assignment.

- *Assistant Mentor, Undergraduate Research Programs*, George Mason University, 2009-2012
  Assisted my advisor, Timothy Sauer, in mentoring undergraduate research students. Participated in weekly meetings, presented special topics which motivated research directions and helped guide the students' research projects. Worked with students both one-on-one and in small groups to read papers, replicate algorithms and results, and identify and explore promising new directions of research or applications. A few of the projects I helped mentor include:
  - **Data Science/Paleoclimatology:** Re-analyzed the standard tree ring data set with novel nonlinear methods and incorporated ice core data and measured atmospheric CO_2 data into the regression. Student was awarded a NSF graduate fellowship and is currently a graduate student.
  - **Data Science/Dynamical Systems:** Developed a method of building better metrics for comparing videos in delay-embedding coordinates based on analysis of the space of sub-images. Student is currently applying to graduate schools.
  - **Inverse Problems/Economics:** Compared stock and options price data in order to infer an implied utility function for various markets. Student is currently a graduate student and received an honorable mention in the NSF graduate fellowship competition.

- *Seminar Instructor, Undergraduate Research Programs*, George Mason University, 2009-2012
  Assisted in multiple Research Experience for Undergraduates (REU) and Undergraduate Research in Computational Mathematics (URCM) Summer Programs, providing lectures on:
  - Introduction to Stochastic Differential Equations and Applications to Portfolio Theory

- Introduction to Dynamical Systems and Attractors
- Introduction to LaTeX

Computer Science Experience

- MATLAB, JAVA, PYTHON, and C/C++ including CUDA programming for Graphics Processing Units (GPUs) and MEX for integration with MATLAB.

Invited Talks

- T. Berry, *Adaptive ensemble Kalman filtering of nonlinear systems.* Applied Dynamics Seminar Series - University of Maryland, Sept. 29, 2016.
`http://math.gmu.edu/~berry/Presentations/umdQR.pdff`

- T. Berry, *Filtering without a model or with a partial model.* 2016 AIMS Conference - Orlando, FL, July 2, 2016.
`http://math.gmu.edu/~berry/Presentations/FilteringAIMS.pdf`

- T. Berry, *Data-driven forecasting without a model and with a partially known model.* 2015 AMMCS-CAIMS Congress - Wilfrid Laurier University, June 11, 2015.
`http://math.gmu.edu/~berry/Presentations/2015AMMCS.pdf`

- T. Berry, *Data-driven forecasting without a model and with a partially known model.* Courant Institute Applied Math Seminar, March 13, 2015.
`http://math.gmu.edu/~berry/Presentations/2015NYU.pdf`

- T. Berry, *Mathematical Theory for Filtering with Model Errors.* SIAM Conference on Uncertainty Quantification, Savannah, GA, April 1, 2014.

- T. Berry, *Linear Theory for Filtering Nonlinear Multiscale Systems with Model Error.* ONR-MURI Workshop, Courant Institute, January 20-22, 2014.
`http://personal.psu.edu/thb11/BerryNYU2014.pdf`

- T. Berry, *Linear Theory for Filtering Nonlinear Multiscale Systems with Model Error.* 4th Annual Joint PSU-UMD Data Assimilation Workshop, Pennsylvania State University, Dec. 18, 2013.
`http://personal.psu.edu/thb11/berrytalkPSU2013.pdf`

- T. Berry, *Adaptive ensemble Kalman filtering of nonlinear systems.* 2013 Interdisciplinary Summer School: Data Assimilation in Geosciences, University of Maryland, College Park, June 3-14, 2013.
`http://www.cscamm.umd.edu/tutorials/`

- T. Berry, *Diffusion Mapped Delay Coordinates and the Geometry of Dynamical Data.* Snowbird SIAM Conference on Applications of Dynamical Systems, May 19-23, 2013.
`http://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=16082`

- T. Berry, *Convergence of periodically forced rank-type equations.* Joint Mathematics Meeting at Boston, January 4, 2012, Special Session for Difference Equations and Applications, I.
`http://jointmathematicsmeetings.org/amsmtgs/2138_abstracts/1077-39-669.pdf`

- T. Berry, *Convergence of periodically forced rank-type equations.* AMS Meeting at Syracuse University, October 2, 2010, Special Session on Difference Equations and Applications.
`http://www.ams.org/meetings/sectional/1062-39-9.pdf`