Math 689: The Mathematics of Manifold Learning
Spring 2019

1 General Comments

The main goal of the course will be to understand the "geometric prior" which is a way of understanding data by assuming that the data lies on/near a sub-manifold of Euclidean space. The focus will be on algorithms that have proven to be effective; explaining mathematically why they work and forming more rigorous assumptions for their application. The course will introduce many concepts from differential geometry and overview key theorems which have implications for how we implement or interpret algorithms. Homework will be projects that mix implementation of algorithms, testing on real and synthetic data sets, and proving lemmas related to the algorithms. Suggested prerequisites would be numerical analysis (685) and linear analysis (675), or instructor permission (a reasonable background in numerics and real analysis should suffice).

- **Instructor:** Tyrus Berry, tberry@gmu.edu, `http://math.gmu.edu/~berry/`
- **Office:** Exploratory Hall, room 4452
- **Office Hours:** TR 2pm-3pm, and by appt.
- **Book:** We will be using chapters from a book-in-progress that I am writing with Dr. Sauer.
- **Topics:** The basic outline for the course will be:
  1. Linear dimensionality reduction - week 1
     - PCA/MDS, Sard's Theorem
  2. Nonlinear dimensionality reduction - week 2
     - ISOMAP, Intro to Manifolds and geodesics, Kernel PCA, Mercer's Theorem
  3. Kernel regression and Reproducing Kernel Hilbert Spaces (RKHS) - week 3
     - Positive definite kernels, Moore-Aronszajn Theorem
  4. Introduction to Riemannian Geometry - weeks 4-6
     - Riemannian metric, normal coordinates, Laplace-Beltrami operator, curvature
  5. The Diffusion Maps theorem - weeks 7-8
     - Proof that certain graph Laplacians converge to Laplace-Beltrami operators on manifolds
  6. Applications of Diffusion Maps - weeks 9-10
     - Representing diffeomorphisms and operators, the diffusion forecast
  7. Persistent homology - weeks 11-12
     - Homology of simplicial complexes, persistence diagrams for homology and computation
  8. Further topics - weeks 13-14
     - Spectral Exterior Calculus (time permitting)
- **Grading:** Grades will be based on homework projects that will include theoretical questions, calculations, and extensive programming assignments (implement/test/verify/apply algorithms from class).