Bifurcation Theory

Math 689-001

Spring 2018

This is the web page **http://math.cos.gmu.edu/~wanner/courses/m689s18/index.html**
It will be updated regularly and always contain the latest information on the course.

General Information:

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Office hours:	TR 1pm-2pm, and by appointment

Lectures:	TR 10:30-11:45am, Exploratory Hall 4106
Prerequisites:	A thorough knowledge of undergraduate advanced calculus, linear algebra, and differential equations is assumed. Also, familiarity with the "Theorem-Proof" style of presentation is important.
Textbook:	No required textbook.

Important Links:

- Detailed syllabus (including recommended books)
- Relevant official GMU policies
- Homework assignments will be posted on Blackboard.

Syllabus:

The course introduces basic concepts and techniques of bifurcation theory for discrete and continuous dynamical systems. These include topological equivalence and structural stability of dynamical systems, local bifurcations of equilibria and periodic orbits, normal forms, and global bifurcations. Time permitting, we will also discuss bifurcations in symmetric systems, as well as bifurcation theory geared towards computer-assisted proofs. At least the beginning of the course will follow the book by Yuri Kuznetsov, *Elements of Applied Bifurcation Theory*, Springer-Verlag. A more detailed syllabus can be found here. It will be updated weekly.

Homework Assignments:

Homework problems will be assigned once a week and posted on Blackboard in the Assignments section. Some of these assignments will be graded and count towards your homework score. While the remaining ones do not have to be handed in, I do advise everyone strongly to study them and write out the solutions properly. I will go through many of the homework problems in the following class and you will not benefit from this if you have not made a serious attempt at solving them.

Grading Policy:

Your final grade in the course will be determined from your performance in the homework assignments, a final project/presentation, and your attendance and class participation. Weights for these items will be distributed approximately according to the following schedule:

Homework	Final Project/Presentation	Attendance
60%	30%	10%

Thomas Wanner, December 12, 2017.

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The following table contains the schedule for the course. This page will be updated regularly throughout the semester.

Week	Date		Sections
		I. Introduction to Dynamical Systems	
1	01/23	1. Continuous and Discrete Dynamical Systems	
		2. Orbits and Phase Portraits	
	01/25	3. Invariant Sets	
2	**01/30**	**No class!**	
	02/01	**No class!**	
3	02/06	4. Differential Equations as Dynamical Systems	
	02/08	5. Poincare Maps	
		II. Topological Equivalence, Bifurcations, and Structural Stability	
		1. Equivalence of Dynamical Systems	
		2. Topological Classification of Equilibria and Fixed Points	
		3. Bifurcations and Bifurcation Diagrams	
		4. Topological Normal Forms	
		5. Structural Stability	
		III. Local Bifurcations in Continuous Dynamical Systems	
		1. Simplest Bifurcation Conditions	
		2. The Fold Bifurcation	
		3. The Hopf Bifurcation	
		4. Center Manifolds	
		5. The Lyapunov-Schmidt Reduction	
		IV. Local Bifurcations in Discrete Dynamical Systems	
		1. Simplest Bifurcation Conditions	
		2. The Fold Bifurcation	
		3. The Flip Bifurcation	
		4. The Neimark-Sacker Bifurcation	
		5. Computation of Center Manifolds	
		V. Bifurcations in Symmetric Systems	
		1. Equivariant Dynamical Systems	
		2. Equivariant Lyapunov-Schmidt Reduction	
		3. Symmetry-Breaking Pitchfork Bifurcations	

VI. Global Bifurcations

1. To be announced..

VII. An Introduction to Computer-Assisted Proofs

1. To be announced..

For the course, I will draw material from the following books:

- Shui-Nee Chow and Jack Hale, *Methods of Bifurcation Theory*, Springer, 1982.
- Jack Hale and Huseyin Kocak, *Dynamics and Bifurcations*, Springer, 1991.
- Yuri Kuznetsov, *Elements of Applied Bifurcation Theory*, Springer, 2004 (3rd edition).

Thomas Wanner, December 12, 2017.

Relevant George Mason
Official University Policies

The following policies apply to all courses at George Mason University:

1. It is expected that each student will conduct himself or herself within the guidelines of the Honor Code. All academic work should be done with the level of honesty and integrity that this University demands.
2. You are responsible for the accuracy of your own schedule. Check Patriot Web regularly to verify that you are registered for the classes that you think you are. A student who is not registered may not continue to attend class. Faculty are not permitted to grade work of students who do not appear on the official class roster.
3. You are responsible for knowing the last days to drop and add this class.
4. Once the add and drop deadlines have passed, instructors do not have the authority to approve any requests from students to add or drop/withdraw late. It is NOT permissible to drop the class and leave it at that. It needs approval. Late adds (up until the last day of classes) must be reviewed and approved by the department chair of the course being offered. They will be approved only in the case of a documented university error (such as a problem with Financial Aid being processed). All student requests for withdrawals and retroactive adds (after the last day of classes) must be reviewed by the student's academic dean. In the case of students whose major is in COS, this is the office of Undergraduate Academic Affairs in Enterprise.
5. Instructors are required to give the final exam at the time and place published in the Schedule of Classes, as set by the Registrar. It cannot be changed. You need to plan vacation (make plane reservations, etc.) around these published dates.
6. Once final grades have been recorded, instructors cannot accept any work to change that course grade. Grade changes can only be approved when they are due to a calculation or recording error on the part of the instructor.
7. An IN (incomplete) grade is a very special grade that can only be applied for in writing. It can only be given in cases in which a student is passing a course and has a very limited amount of work left to complete the course.
8. Federal law (a law known as FERPA) requires the protection of privacy of student information. Therefore, no instructor on campus can speak about a student's record with anyone other than the student. The record includes how a student is doing in a course, whether a student has attended class, information about grades, whether a paper has been turned in. Anything. This prohibition includes parents, siblings, and spouses, anyone.
