# Generalized Graphic Matroids in Projective Geometry

Thomas Zaslavsky (Binghamton University, SUNY)

A graph $\Gamma$ has a vector representation in which an edge $e_{ij} = v_i v_j$ corresponds to the vector $b_j - b_i$, where $b_1, b_2, \ldots$ are the standard unit basis vectors. Such a representation is a vector representation of the graphic matroid $M(\Gamma)$. We may restate this as a mapping of $E(\Gamma)$ into projective space, giving a projective representation of $M(\Gamma)$. Properties of graphs imply that this representation is unique up to projective transformations.

A gain graph $\Phi$ is a graph with something more: an orientable "gain function" $\varphi : E \to \mathfrak{H}$ where $\mathfrak{H}$ is a group. $\Phi$ has a "frame matroid" $M(\Phi)$ that generalizes the graphic matroid. If $\mathfrak{H} \leq F^\times$, the multiplicative group of a field, then $\Phi$ has a vector representation $(e, g) \mapsto b_j - \varphi(e_{ij}) b_i$ that represents the frame matroid. We can restate this as a representation of $M(\Phi)$ in projective space (which may not be unique).

What if $\mathfrak{H}$ is not in any $F^\times$? Even then we may be able to represent $M(\Phi)$ projectively by using synthetic geometry. I will explain how this works, first in any dimension, and then for rank-3 matroids in projective planes. The special feature of the latter is that, while all higher-dimensional projective spaces have coordinates in a field (or skew field), most projective planes do not; this means one needs a more combinatorial treatment of representation involving quasigroups and ternary rings, which I will explain.

This report is on joint work with Rigoberto Flórez.