Shearer's Method in Ramsey Theory

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Abstract

In Ramsey Theory one is interested in constructing graphs or hypergraphs with no large independent sets and which lack certain subgraphs. It is well-known that good constructions arise by considering random graphs. This includes the semirandom construction of n-vertex triangle-free graphs whose independent sets have size $O(\sqrt{n \log n})$ of Kim and later Bohman-Keevash and Fiz Pontiveros-Griffiths-Morris. A remarkable approach with the use of conditional expectation was given by Shearer, showing that this bound is tight up to constants. In this talk we outline the latter approach, as well as recent advances on extending the method to other Ramsey theoretic problems. In particular, improving earlier results of Ajtai-Komlós-Szemerédi and Rödl-Sijanova, we obtain near-optimal bounds on the size of the largest independent set in a Steiner r-tuple system. In conclusion, we present some open problems.

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