OPERATOR THEORY ON THE BLOCH SPACE

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In recent years, the operator theory of many functional Banach spaces that arise in complex function theory has been studied extensively. In this talk, two important classes of operators will be discussed: the *composition operators*

$$C_\varphi(f) = f \circ \varphi$$

and the *multiplication operators*

$$M_\psi(f) = \psi f,$$

where φ and ψ are fixed functions defined on a region D called the *symbols* of the operators C_φ and M_ψ, and f belongs to some functional Banach space with domain D. An environment for this study is a space of analytic functions on the open unit disk $\mathbb{D}$ called the *Bloch space*.

An analytic function f defined on the open unit disk $\mathbb{D}$ is said to be *Bloch* if

$$\beta_f := \sup_{z \in \mathbb{D}} (1 - |z|^2)|f'(z)| < \infty.$$

The set $\mathcal{B}$ of Bloch functions is a Banach space under the norm $\|f\|_{\mathcal{B}} = |f(0)| + \beta_f$. Characterizations on the boundedness and compactness of the operators C_φ and M_ψ as well as operator norm estimates and a description of the spectrum will be given.

We conclude the talk with a status report on the composition and multiplication operators on the Bloch space of a bounded homogeneous domain in $\mathbb{C}^n$. This is joint work with Robert F. Allen.