An Introduction to Matroid Theory Through Lattice Paths

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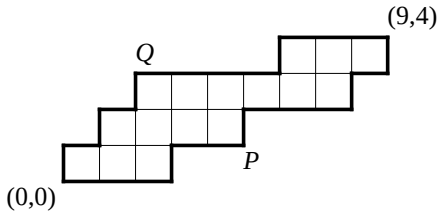

For fixed lattice paths P and Q that go from $(0,0)$ to (m,r), with P never going above Q, how many lattice paths from $(0,0)$ to (m,r) stay in the region that P and Q bound? Such enumerative questions have been studied extensively. We develop the theory of lattice paths in a new, more structural direction, showing that the paths that remain in the region bounded by P and Q can be identified with the bases of a special type of matroid — a lattice path matroid. We use these very accessible examples to introduce some of the basic ideas of matroid theory. Many important invariants (such as the Tutte polynomial) that are $\#P$-hard to compute for arbitrary matroids have natural interpretations for lattice path matroids, and these interpretations yield polynomial-time algorithms for computing the invariants for this special class of matroids. Also, we describe how the matroid perspective on lattice paths leads to new enumerative results.

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