Counting Faces of Polytopes

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A convex polytope in $\mathbf{R}^d$ is the smallest convex set containing a given finite set of points in $\mathbf{R}^d$. Familiar three-dimensional examples are the Platonic solids. Of long interest are the numbers of faces of different dimensions that polytopes can have—starting, perhaps, with Euler's relation $V - E + F = 2$, and its generalizations to higher dimensions. I will talk about some of the interplay between discrete geometry, combinatorics, commutative algebra, and algebraic geometry in approaching such problems, including some interesting connections with stress on bar-and-joint frameworks, and formulas for volumes of polytopes.