Department of Mathematical Sciences George Mason University
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Topology Preliminary Exam

This exam consists of 6 questions.

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1. Prove that every compact Hausdorff space is regular.
2. Prove that a second countable topological space is Lindelöf.
3. Let $(\Omega, \prec)$ is the first uncountable ordinal with the order topology.
   (a) Prove that $(\Omega, \prec)$ is not compact.
   (b) Prove that $(\Omega, \prec)$ is limit point compact.
4. Prove that the Cantor Set, defined by the middle-third construction, is homeomorphic to $\{0,1\}^\omega$.
5. Suppose that for each $\alpha \in I$, (X_α, τ_α) is a topological space. Show that if $\prod_{\alpha\in I} X_\alpha$ is Hausdorff, then each space X_α is Hausdorff.
6. Suppose that (X, σ) and (Y, τ) are topological spaces such that (Y, τ) is Hausdorff, and $f, g\colon X \to Y$ are continuous functions. Prove that $\{x : f(x) = g(x)\}$ is closed in X.