Name: ______________________________

Topology Preliminary Exam January 15, 2019

Note: Any space referred to in this exam is both nonempty and Hausdorff, with the possible exception of the last problem.

1. Let X and Y be connected spaces and let $X \times Y$ have the product topology. Prove that $X \times Y$ is connected.
2. Prove that $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^2$.
3. Let $\mathbb{D}$ be the unit disk in $\mathbb{R}^2$ and let $X = \overline{\mathbb{D}}/\partial\mathbb{D}$ be the quotient space of the closure of X by the boundary of X. Prove that X is homeomorphic to the two-sphere $\mathbb{S}^2$.
4. Endow both $\mathbb{N} \times \{0, 1\}$ and $\{0, 1\} \times \mathbb{N}$ with the lexicographic order topologies. One of these two spaces is discrete and the other is not. Decide which is which and show why.
5. In the Zariski topology on the complex plane $\mathbb{C}$, a subset $A \subset \mathbb{C}$ is closed if and only if it is the zero set of a polynomial with complex coefficients. Prove that the Zariski topology is, in fact, a topology. Is it Hausdorff?