Preliminary Exam 2

1. Prove that every compact Hausdorff space is normal.

2. Prove that the continuous image of a compact space is compact.

3. Suppose that for each $\alpha \in I$, (X_α, τ_α) is a topological space and $A_\alpha \subseteq X_\alpha$. Show that $\prod_{\alpha \in I} A_\alpha$ is closed in $\prod_{\alpha \in I} X_\alpha$ in the Tychonoff product topology if and only if A_α is closed in X_α for each $\alpha \in I$.

4. Prove that any second-countable topological space is separable. Give an example of a separable space which is not second-countable.

5. Suppose that (X, d) is a Cauchy complete metric space and $\{F_i\}_{i \in \mathbb{N}}$ is a collection of closed subsets such that for each $i \in \mathbb{N}$, $F_{i+1} \subseteq F_i$ and $\lim_{n \to \infty} diam(F_i) = 0$. Prove that $\cap_{i \in \mathbb{N}} F_i \neq \emptyset$.

6. Prove that the continuous image of a connected space is connected.