Department of Mathematical Sciences George Mason University

Topology Preliminary Exam, August 17, 2010

Throughout this exam $\mathbb{R}$ denotes the set of real numbers with the metric d where $d(x,y) = |x-y|$:, $\mathbb{P}$ denotes the set of irrational numbers, and $\mathbb{C}$ denotes the Cantor Set. Metric spaces are assumed to have the metric topology, and subsets of topological spaces are assumed to have the subspace topology. Also, $\mathbb{R}_L$ denotes the Sorgenfrey Line (with topology generated by half open intervals), and the symbol $S(\Omega)$ refers to the space of countable ordinals with the natural order topology.

This exam consists of 6 questions.

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1. Prove that no two of these subspaces of $\mathbb{R}$ are homeomorphic: $[0,1]$, $\mathbb{P}$, $\mathbb{C}$.
2. Let (X,T) be a normal topological space and let (D,T^*) be a discrete topological space. Prove that the product $X \times T$, with the product topology, is a normal topological space.
3. Prove that $\mathbb{R}_L$ is not metrizable.
4. Suppose that A and B are closed subsets of $S(\Omega)$, and $A \cap B = \emptyset$. Show that at least one of A and B is bounded in $S(\Omega)$.
5. If U and V are nonempty basic product topology open sets for $\{0,1\}^J$, where J is an infinite set, prove that U and V are homeomorphic subspaces of $\{0,1\}^J$.
6. Let $[0,1]^2$ be endowed with the lexicographic order topology. Show that any connected subset which is separable must be homeomorphic to an interval of $\mathbb{R}$.
