Throughout this exam \( \mathbb{R} \) denotes the set of real numbers with the metric \( d \) where \( d(x, y) = |x - y| \); \( \mathbb{P} \) denotes the set of irrational numbers, and \( \mathbb{C} \) denotes the Cantor Set. Metric spaces are assumed to have the metric topology, and subsets of topological spaces are assumed to have the subspace topology. Also, \( \mathbb{R}_L \) denotes the Sorgenfrey Line (with topology generated by half open intervals), and the symbol \( S(\Omega) \) refers to the space of countable ordinals with the natural order topology.

This exam consists of 6 questions.

1. Prove that no two of these subspaces of \( \mathbb{R} \) are homeomorphic: \([0, 1], \mathbb{P}, \mathbb{C}\).

2. Let \((X, T)\) be a normal topological space and let \((D, T^*)\) be a discrete topological space. Prove that the product \( X \times T \), with the product topology, is a normal topological space.

3. Prove that \( \mathbb{R}_L \) is not metrizable.

4. Suppose that \( A \) and \( B \) are closed subsets of \( S(\Omega) \), and \( A \cap B = \emptyset \). Show that at least one of \( A \) and \( B \) is bounded in \( S(\Omega) \).

5. If \( U \) and \( V \) are nonempty basic product topology open sets for \( \{0, 1\}^J \), where \( J \) is an infinite set, prove that \( U \) and \( V \) are homeomorphic subspaces of \( \{0, 1\}^J \).

6. Let \([0, 1]^2\) be endowed with the lexicographic order topology. Show that any connected subset which is separable must be homeomorphic to an interval of \( \mathbb{R} \).