Department of Mathematical Sciences George Mason University

Topology Preliminary Exam, August, 2011

Throughout this exam $\mathbb{R}$ denotes the set of real numbers with the metric d where $d(x, y) = |x-y|$:, $\mathbb{P}$ denotes the set of irrational numbers, and $\mathbb{C}$ denotes the Cantor Set. Metric spaces are assumed to have the metric topology, and subsets of topological spaces are assumed to have the subspace topology. Also, $\mathbb{R}_L$ denotes the Sorgenfrey Line (with topology generated by half open intervals), and the symbol $S(\Omega)$ refers to the space of countable ordinals with the natural order topology.

This exam consists of 6 questions.

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1. Prove that any bounded and non-empty open set in $\mathbb{R}^2$ has infinitely many boundary points.
2. Let (X, T) be a Hausdorff topological space and let (Y, T^*) be a normal topological space. Prove that the product $X \times Y$, with the product topology, is a Hausdorff topological space.
3. Prove that $\mathbb{R}_L$ does not have a countable basis.
4. Suppose that A is an infinite subset of $S(\Omega)$ Prove that A has a limit point in $S(\Omega)$.
5. If Y is a connected and dense subset in (X, T), prove that (X, T) is a connected topological space..
6. Let $(X, <_x)$ and $(Y, <_Y)$ be well ordered sets. Prove that $X \times Y$ with the lexicographical order is a well ordered set..
