# Topology Preliminary Exam

This exam consists of 6 questions. The following notation will be used throughout this exam: $\mathbb{R}$ denotes the set of real numbers and it and all of its subspaces have the usual topology, induced by the metric $d$ where $d(x, y) = |x - y|$.

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**(1)** Prove that a metric space is separable if and only if it has a countable base.

**(2)** (a) Prove that the open interval $(0, 1)$ in $\mathbb{R}$ is homeomorphic to $\mathbb{R}$.

(b) Prove that the half-open interval $[0, 1)$ is not homeomorphic to $\mathbb{R}$.

**(3)** Prove that if $X$ and $Y$ are Hausdorff spaces, then the product $X \times Y$ is a Hausdorff space.

**(4)** Prove that a compact Hausdorff space is normal.

**(5)** Suppose that $A$ and $B$ are connected subspaces of a space $X$ such that $A \cap B \neq \emptyset$. Prove that $A \cup B$ is connected.

**(6)** Prove that the product space $\{0, 1\}^\omega$ is not discrete.

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