TOPOLOGY PRELIMINARY EXAM
SPRING 2015

Problem 1

Prove that the spaces $(0,1)$, $[0,1)$ and S^1 (the circle) are mutually non-isomorphic.

Give an example of a continuous, bijective map which is not a homeomorphism.

Problem 2

Let X and Y be path-connected and simply connected. Prove that $X \times Y$ is path-connected and simply connected.

Problem 3

Let X be a compact Hausdorff space, prove that X is a regular space.

Problem 4

Suppose $f, g : X \to Y$ are continuous maps, and Y is a Hausdorff. Prove that the set $f = g$ is closed.

Problem 5

Prove that any bounded, non-empty subset of the complex numbers $\mathbb{C}$ has infinitely many boundary points.

Problem 6

Let X be a locally compact Hausdorff space. Define what it means for a space $X \subset \bar{X}$ containing X to be a compactification of X. Prove that $\bar{X} \setminus X$ is compact.