Department of Mathematical Sciences George Mason University

# Topology Preliminary Exam, August 2019

This exam consists of 6 questions, all of equal weight. You must do *only* 5 questions. DO NOT submit solutions to all 6 problems.

---

**(1)** Prove that a locally compact Hausdorff space is completely regular.

**(2)** Let $X$ be path-connected. Prove that $\pi_1(X)$ is abelian if and only if all basepoint-change homomorphisms $\beta_h$ depend only on the endpoints of the path $h$.

**(3)** For a collection of spaces $X_\alpha$, $\alpha \in \mathcal{A}$, let $\prod_\alpha X_\alpha$ be the cartesian product with the product topology. Let $X$ be a topological space. Show that a function $f : X \to \prod_\alpha X_\alpha$ is continuous if and only if the composition $X \to \prod_\alpha X_\alpha \to X_\beta$ is continuous for each $\beta \in \mathcal{A}$ where $\prod_\alpha X_\alpha \to X_\beta$ is the canonical projection $\{x_\alpha\}_\alpha \mapsto x_\beta$.

**(4)** Show that a closed map $f : X \to Y$ is proper if each fibre $f^{-1}(y)$ is compact.

**(5)** Prove that if $A$ is a connected subset of a topological space $X$ and $A \subset B \subset \overline{A}$, then $B$ is connected.

**(6)** Prove that a metric space is separable if and only if it is second countable.

---