Topology preliminary Exam
August 20, 2013

The space of real numbers is denoted $\mathbb{R}$, and $\mathbb{R}_L$ denotes the same set with the Sorgenfrey line topology. $S(\Omega)$ denotes the set of countable ordinals with the order topology. $S(\Omega)$ is uncountable.

1. Let A and B be nonempty subsets of $\mathbb{R}^2$.
   (a) Give an example to show that $A \cup B$ can be connected while $A \cap B = \emptyset$ and neither A nor B is connected.
   (b) Prove that if A and B are both closed and $A \cup B$ is connected, then $A \cap B \neq \emptyset$.
2. Let C be a collection of non-degenerate circles in $\mathbb{R}^2$ souch that no two intersect. Show that $\mathbb{R}^2 - \cup C \neq \emptyset$
3. Show that if $M = [0, 1)$ and $N = (0, 1)$ are viewed as subspaces of $\mathbb{R}_L$, the Sorgenfrey line, then M and N are homeomorphic by accurately describing a homeomorphism. You do not need to actually prove it is a homeomorphism.
4. Let L denote the set of limit points in $S(\Omega)$, using the order topology. If $H \subset S(\Omega)$ is uncountable and closed, then $H \cap L \neq \emptyset$
5. Let $P = \{0, 1\}^S$ where S is an uncountable set, and suppose P has the product topology. Suppose $f : P \to [0, 1]$ is continuous and $\sigma \in P$ with $f(\sigma) = 1$. Show there is at least one other point $\tau \in P$ with $\tau \neq \sigma$ and such that $f(\tau) = 1$.