Linear Analysis Preliminary Exam August 2015

Instructions: This exam is closed book, closed notes, no calculator or other electronic device. Do all of the following four questions.

1. Precisely state the Hahn-Banach theorem for real linear spaces. Prove the result in as much detail as you can paying particular attention to the application of Zorn's Lemma.

2. (a) State the Heine-Borel property.

   (b) Give an example of a space that does not have the Heine-Borel property, and prove that this space does not have the Heine-Borel property.

3. Let H be a real Hilbert space. Prove the Riesz Representation Theorem (which describes all continuous linear functionals on H) and then explain how we may then view H^*, the dual space to H, as being isomorphic to H.

4. Consider the Banach space $C[-1,1]$ and the linear mapping $L : C[-1,1] \to C[-1,1]$ given by $(Lf)(x) = 2x\, f(x)$. Find the norm of L and also find its spectrum, meaning those values of λ for which $\lambda I - L$ fails to have a bounded inverse.