Department of Mathematical Sciences George Mason University

Algebra Preliminary Exam, August 2012

This exam consists of 5 questions.

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1. Let G be a group and $H \subseteq G$ a subgroup. Then $N_G(H) = \{g \in G : gH = Hg\}$ is the *normalizer of H in G*.
   (a) Show that $N_G(H)$ is the largest subgroup of G in which H is normal.
   (b) If $H_1 \subseteq H_2$, what is the relations between $N_G(H_1)$ and $N_G(H_2)$?
2. How many elements of order 11 are there in a group of order 693?
3. let R be a ring given by
$$R = \left\{ \begin{pmatrix} n & m \\ m & m+n \end{pmatrix} : n, m \in \mathbb{Z}_5 \right\},$$
with the usual matrix addition and multiplication. Prove that R is a finite commutative ring. How many elements does R have? Determine the units of R.
4. Show that any finite multiplicative subgroup of the complex field $\mathbb{C}$ is cyclic.
5. Let $a = \sqrt{2} + \sqrt[3]{3} + \sqrt[5]{5} \in \mathbb{R}$. Show that a is algebraic over $\mathbb{Q}$ and that $[\mathbb{Q}(a) : \mathbb{Q}] \leq 30$.
