Topology Preliminary Exam
August, 2012

This exam consists of 8 questions.

With Proof

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(1) Show that if X is a compact Hausdorff space, then X is normal..

(2) Show that a well ordered space $\langle X, < \rangle$ under the order topology is compact if and only if it has a largest element with regard to $<$.

(3) Give an example of a totally ordered space $\langle X, < \rangle$ which under the order topology is sequentially compact but not compact.

(4) Show that if X is a topological space with connected subspaces A, B, where $A \cap B \neq \emptyset$ and $X = A \cup B$, then X is connected.

(5) Show that for a compact space X and a Hausdorff space Y, a continuous function $f : X \to Y$ is a homeomorphism if and only if it is a bijection.

Without Proof

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(6) Give an example of a complete metric space $\langle X, d \rangle$ having no isolated points, that is non-compact where for all $x, y \in X$, $d(x, y) \leq 1$.

(7) Give an example of a normal space X where $X \times X$ is non-normal. Also give either two disjoint closed sets that witness the non-normality of $X \times X$ or justify why Jones lemma applies.

(8) Determine which of the following are true about a metric space X.

- (a) If X is separable, then X is Lindelöf.
- (b) The space X is compact if and only if it is sequentially compact.
- (c) If X is Lindelöf, then X is second countable.
- (d) The space X is second countable if and only if it is separable.
- (e) The space X is limit point compact if and only if it is sequentially compact.
- (f) If X is compact, then X is separable.
