

An Approach to Deterministic Optimal Control Synthesis Based on the Generalized Method of Characteristics

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Even for sufficiently simple single-input control-affine systems without state constraints the subject of analytical and qualitative investigation of global optimal control synthesis is developed rather poorly in the literature on optimal control theory and its applications. This talk presents numerical-analytical and qualitative methods to construct global optimal control synthesis with reference to a nonlinear mathematical model describing interactions between the production of a company, its technology stock and R&D investments on a fixed time interval. Here the control is the R&D investments' intensity. The problem is to maximize the production at a fixed time horizon. These methods use the extension of classical Cauchy's characteristics method for first-order partial differential equations. An important feature of the developed approach is a simultaneous use of the following two types of qualitative data:

- analytical representations providing local solutions of the Cauchy problem for the Hamilton-Jacobi-Bellman equation, which correspond to boundary constant controls;
- the results of bang-bang and singular controls' global investigation by Pontryagin's maximum principle.

Hence, dynamic programming method (sufficient optimality conditions) is combined with Pontryagin's maximum principle (necessary optimality conditions). Furthermore, the stated approach is related to the cumulative experience acquired during the construction of optimal control synthesis for some biomedical and mechanical mathematical models. This talk gives the description of all possible geometric portraits for global optimal control synthesis in the considered mathematical model of economic growth under R&D investments. Some sufficient conditions for the smoothness of the value function are also introduced.

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