3.2.8. Let \( w \) be the portion of the $1000 that is invested in the risky asset; then \( 1000 - w \) is put into the bond. The yield, in dollars, is then
\[
Y = (1000 - w)(.04) + w(.01)R
\]
because \( R \) is in units of cents. Using the graph, you can find that the density of \( R \) is
\[
f(x) = \begin{cases} 
\frac{1}{6}(x - 3) & \text{if } x \in [3, 5) \\
\frac{1}{12}(x - 9) & \text{if } x \in [5, 9] 
\end{cases}
\]
The expectation of \( R \) can be found in a straightforward way:
\[
E[R] = \int_{3}^{5} x \cdot \frac{1}{6}(x - 3) \, dx + \int_{5}^{9} x \cdot (-\frac{1}{12}(x - 9)) \, dx = 17/3.
\]
\[
E[R^2] \text{ can be found similarly:}
\]
\[
E[R^2] = \int_{3}^{5} x^2 \cdot \frac{1}{6}(x - 3) \, dx + \int_{5}^{9} x^2 \cdot (-\frac{1}{12}(x - 9)) \, dx = 101/3.
\]

Therefore \( \sigma^2 = \text{Var}(R) = 101/3 - (17/3)^2 = 14/9 \).

The mean of \( Y \) minus half of the variance of \( Y \) is
\[
E[Y] - \frac{1}{2}\text{Var}(Y) = E[(1000 - w)(.04) + w(.01)R] - \frac{1}{2}\text{Var}(w(.01)R) = (1000 - w)(.04) + w(.01) \cdot \frac{17}{3} - \frac{1}{2}(.01)^2w^2 \cdot \frac{14}{9} = 40 + \frac{1}{60}w - \frac{7}{90000}w^2.
\]

Setting the derivative of the last function with respect to \( w \) equal to 0 produces the critical point \( w = 107.14 \) for the amount to invest in the stock, and therefore $892.86 in the bond.

**Note that if you express \( Y \) in cents rather than dollars:**
\[
g(w) = E(Y) - \frac{1}{2}\text{Var}(Y) = (1000 - w)4 + w \cdot \frac{17}{3} - \frac{1}{2}w^2 \cdot \frac{14}{9}.
\]
\[
g'(w) = -4 + \frac{17}{3} - w \cdot \frac{14}{9}
\]
\[
g'(w) = 0 \text{ for } w = 107.14.
\]
This seems optimal strategy depends on unit.

Does that seem reasonable?
3.2.6. From Exercise 3.1.18, the c.d.f. is

\[ F(t) = 1 - (e^{-0.02t} + .02te^{-0.02t}). \]

Its derivative, the density function, is

\[ f(t) = .02e^{-0.02t} - .02e^{-0.02t} + (.02)^2te^{-0.02t} = (.02)^2te^{-0.02t}. \]

Thus, the expected value of the service time is

\[
E[T] = \int_0^\infty (.02)^2t^2e^{-0.02t} \, dt \\
= .02t^2e^{-0.02t}\bigg|_0^\infty + 2 \int_0^\infty .02te^{-0.02t} \, dt \\
= 0 + 2 \cdot \frac{1}{.02} = 100.
\]

The second line uses integration by parts, and the third line uses the result in Exercise 3.2.1.

3.2.12.

\[
E[X_1 + X_2] = \int_0^\infty \int_0^\infty (x_1 + x_2) \cdot \frac{1}{2} \, dx_2 \, dx_1 \\
= \frac{1}{2} \int_0^\infty (x_1^2 + x_2^2) dx_1 \\
= \frac{3}{4} \int_0^\infty x_1^2 \, dx_1 \\
= 2.
\]

3.3.1. (a) \( T_1 \) has the \( \exp(1) \) distribution, hence

\[ P[T_1 > 3] = \int_3^\infty e^{-t} \, dt = e^{-3}. \]

(b) \( T_2 \) has the \( \Gamma(2,1) \) distribution, hence

\[ P[T_2 \in (1,3)] = \int_1^3 te^{-t} \, dt = -te^{-t} \bigg|_1^3 = -4e^{-3} + 2e^{-1}. \]

(c) \( E(T_5^{-1}T_1) = E(T_5) - E(T_1) \)

\[ \sim 5^{-1} = \frac{1}{5} \]

3.3.12. (a) Directly from the \( r = 8 \) line of the \( \chi^2 \) table, by complementing, \( x = 15.51. \)

(b) \( P[3.49 < X \leq 13.36] = P[X \leq 13.36] - P[X \leq 3.49] = .9 - .1 = .8. \)

(c) Using the .95 and .05 columns,

\[ P[2.73 < X \leq 15.51] = P[X \leq 15.51] - P[X \leq 2.73] = .95 - .05 = .9. \]

There would be a lot of ways to choose \( c \) and \( d \); just be sure that \( P[X \leq d] - P[X \leq c] = .9. \) We could use the 95th and 6th percentiles for example.