11.5

#8 \( y'' + w^2 y = \frac{3}{x} \left[ \sin (2k+1)x \right] / (2k+1) \); \( y_0 = c_1 \cos wt + c_2 \sin wt \)

Assume \( y_p = \frac{3}{x} B_k \sin (2k+1)x \)

Then \( \sum_{k=0}^{\infty} B_k (w^2 - (2k+1)^2) \sin (2k+1)x = \frac{3}{x} \sin (2k+1)x / (2k+1) \)

Thus \( B_k = \frac{1}{\left[ w^2 - (2k+1)^2 \right] / (2k+1)} \cdot \frac{3}{x} \sin (2k+1)x \)

Note: The cases where \( w = 0, 1, 3, 5, \) or 7 must be treated separately.

#14 Assume \( y_p = A_m \cos nt + B_n \sin nt \)

\[-m^2 A_m \cos nt - m^2 B_n \sin nt + c (-m A_m \cos nt + m B_n \sin nt) \]

\[ + A_m \cos nt + B_n \sin nt = A_m \cos nt \]

\[ (1-m^2) A_m + c m B_n = a_m \quad \text{let} \; \Delta_m = (1-m^2)^2 + m^2 c^2 > 0 \]

\[-c m A_m + (1-m^2) B_n = 0 \quad \text{so be Cramer's rule} \]

\[ A_m = a_m (1-m^2) / \Delta_m \quad \text{and} \quad B_n = a_m c m / \Delta_m \]

#20 \( E'(t) = \sum_{k=0}^{\infty} w \left[ \frac{\cos (2k+1)x}{(2k+1)^2} \right] \)

\( = \sum_{k=0}^{\infty} \frac{\cos (2k+1)x}{(2k+1)^2} \quad \text{let} \; 200 \pi(x) \text{in example 1, or } 11 \text{in 11.3} \)

Assume \( I_p = \sum_{k=0}^{\infty} [A_k \cos (2k+1)x + B_k \sin (2k+1)x] \)

\[ 10 I'' + 100 I' + 100 I = -10 \sum_{k=0}^{\infty} \frac{(2k+1)^2}{(2k+1)^2} [A_k \cos (2k+1)x + B_k \sin (2k+1)x] \]

\[ + 100 \sum_{k=0}^{\infty} (2k+1) \left[ A_k \cos (2k+1)x + B_k \sin (2k+1)x \right] + 100 \sum_{k=0}^{\infty} [A_k \cos (2k+1)x + B_k \sin (2k+1)x] \]

\[ = E' = \sum_{k=0}^{\infty} \frac{\cos (2k+1)x}{(2k+1)^2} \]

\[ (10 - (2k+1)^2) A_k + 10(2k+1) B_k = \frac{80}{\pi (2k+1)^2} \quad \Delta_k = (10 - (2k+1)^2)^2 - \frac{800}{100 (2k+1)^2} \]

\[ - 10(2k+1) A_k + (10 - (2k+1)^2) B_k = 0 \]

\[ A_k = \frac{80(100 - (2k+1)^2)}{\pi (2k+1)^2 \Delta_k}; \quad B_k = \frac{800}{\pi (2k+1)^2 \Delta_k} \]
11.6

#2 By #2 of 11.1, \( f(x) = \frac{\pi^2}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2} \).

\[ S_n f(x) \Delta x = S_n \Delta x^4 \Delta x = \frac{\pi}{3} n^5 \]

\[ E_1 = \frac{\pi}{3} n^5 - \pi (\frac{\pi}{2} n^4 + 16) \approx 4.14 \quad E_2 = E_1 - \pi = 1.00 \; E_3 = E_2 - \frac{16}{625} \approx 0.8 \]

\[ E_4 = E_3 - \frac{\pi}{16} = 0.18 \; E_5 = E_4 - \frac{16 \pi}{625} \approx 0.10 \]

#6 After finding \( \alpha_n = 0 \), \( a_0 = \frac{1}{2} \), \( a_n = \frac{2}{\pi} \), \( b_n = \frac{2}{\pi} \),

\[ a_n = \frac{2}{\pi} \int_0^\pi \left[ 1 + \frac{1}{2} (-1)^n \right] \Delta x = \{1 - \frac{2}{\pi} \} \approx 0.69 \]

\[ E_1 = E_1 - \pi \left( \frac{2}{12} \right) \Delta x \approx 0.022 \; E_2 = E_2 \approx 0.04 \; E_3 = E_3 \approx 0.008 \]

\[ E_4 = E_4 - \frac{1}{12} \Delta x = 0.004 \; E_5 \approx 0.002 \]

#8 By #6 of 11.1, \( f(x) = \frac{3}{2} \int_0^\pi \frac{\cos 2x}{4 (\pi - x)^2} \Delta x = \frac{3}{2} \int_0^\pi \left( \frac{n}{\pi} \right)^2 \Delta x = \frac{3}{2} \times \left( \frac{\pi}{2} \right)^2 = \frac{3}{8} \]

\[ \frac{3}{8} + \frac{1}{2} \int_0^\pi \frac{1}{(2k+1)^4} \Delta x = \frac{1}{2} \int_0^\pi \frac{1}{(2k+1)^4} \Delta x \approx \frac{\pi}{96} \]

#12 By #15 of 11.1, \( f(x) = \frac{\pi}{2} + \frac{\pi}{4} \sum_{k=0}^{\infty} \frac{\cos (2k+1)x}{(2k+1)^2} \)

\[ \frac{1}{\pi} \sum_{k=0}^{\infty} \frac{1}{4 (2k+1)^4} \Delta x = \frac{1}{\pi} \int_0^\pi \left( \pi - x \right)^2 \Delta x = \frac{\pi}{2} \times \left( \frac{\pi}{2} \right)^2 = \frac{\pi^5}{2} \]

\[ 2 \left( \frac{\pi^2}{2} \right)^2 + \frac{1}{16} \int_0^\pi \frac{1}{(2k+1)^4} \Delta x = \frac{1}{16} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^4} \approx \frac{\pi^4}{96} \]

#4 By #5 of 11.6, \( f(x) = \frac{3}{16} \sin 2x \sum_{k=1}^{\infty} \frac{\cos 2kx}{k^2 (2k+1)^2} \)

\[ \frac{1}{\pi} \int_0^\pi f(x) \Delta x = \frac{1}{\pi} \int_0^\pi \sin x \sum_{k=1}^{\infty} \frac{\cos 2kx}{k^2 (2k+1)^2} \Delta x \]

\[ = \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{1}{k^2 (2k+1)^2} \left( \frac{1}{16} \int_0^\pi \left( \pi - \frac{1}{2} \pi \right)^2 \Delta x \right) = \frac{1}{16} \left( \frac{\pi}{2} \right)^2 \]

\[ = 2 \left( \frac{\pi}{2} \right)^2 + \frac{1}{16} \sum_{k=1}^{\infty} \frac{1}{k^2 (2k+1)^2} \]

\[ = \frac{\pi^2}{2} + \frac{1}{16} \sum_{k=1}^{\infty} \frac{1}{k^2 (2k+1)^2} \]

\[ = \frac{\pi^2}{2} - \frac{1}{2} = \frac{\pi^2}{2} - \frac{1}{2} = \frac{\pi^2}{2} - \frac{1}{2} \]