1. Given that the derivative $f'(x) = x^3 - 3x^2$, $f(0) = 2$, find:
   a) the intervals on which $f$ is increasing or decreasing;
   b) the intervals on which $f$ is concave up or concave down;
   c) the $x$ coordinates of relative extrema and inflection points of $f$.
   d) Use the above to sketch a possible graph of $f(x)$.

   a) $f'(x) = x^2(x-3) = 0 \iff x = 0$ or $3$

   $f' \quad - \quad +$
   $f \quad \rightarrow \quad 0 \quad \rightarrow \quad 3 \quad \rightarrow$

   Decreasing for $x < 3$
   Increasing for $3 < x$

   b) $f''(x) = 3x^2 - 6x = 3x(x-2) = 0 \iff x = 0$ or $2$

   $f'' \quad + \quad - \quad +$

   Concave up for $x < 0$ and $2 < x$
   Concave down for $0 < x < 2$

   c) rel min at $x = 3$
   inf points at $x = 0$ or $x = 2$

   

2. Find the function $f(x)$ given above.

   $f(x) = S(x^3 - 3x^2) \Delta x = x^{4/3} - x^3 + C$

   $a = f(0) = C$

   $f(1) = x^{4/3} - x^3 + 2$
3. The total cost of producing q items is \( C(q) = 5 + 4q - \ln(q + 1) \). Find:
   a) the marginal cost;  
   b) an estimate for the cost of producing the 4\(^{th}\) item;  
   c) the actual cost of producing the 4\(^{th}\) item.

   a) \( C'(q) = 4 - 1/(q+1) \)

   b) \( C'(3) = 4 - \frac{1}{4} = \frac{3}{4} \)  
   \[ \frac{3}{4} = 3.75 \]

   c) \( C(4) - C(3) = (5 + 16 - \ln 5) - (5 + 12 - \ln 4) = 4 - \ln (\frac{5}{4}) = \frac{3}{78} \)  
   \[ \frac{3}{78} = 3.78 \]

4. q widgets can be sold at a price of 100 e\(^{-0.1q}\) per widget.  
Find the production for which the revenue is: a) largest;  
               b) smallest.

\[ R(q) = q \cdot 100 \cdot e^{-0.1q} \]

\[ R'(q) = 100 \cdot e^{-0.1q} + q \cdot 100 \cdot e^{-0.1q} \cdot (-0.1) \]

\[ = 100 \cdot e^{-0.1q} (1 - 0.1q) = 0 \iff q = 10 \]

\[ R'(q) = \frac{\partial R}{\partial q} = \begin{cases} + & q < 10 \\ - & q > 10 \end{cases} \]

a) largest for \( q = 10 \)

b) smallest for \( q = 0 \)

5. You invest $1000 at 8\% annual interest compounded quarterly. Find:
   a) the balance after 20 years;  
   b) when the balance will be $2000.

\[ P = 1000; \quad r = 0.08; \quad k = 4; \quad B = 1000 \cdot (1 + 0.02)^{4t} \]

a) \( B = 1000 \cdot (1.02)^{80} = 4875.44 \)

b) \( 2000 = 1000 \cdot (1.02)^{4t} \]

\[ 2 = (1.02)^{4t} \]

\[ \log 2 = 4t \log 1.02 \]

\[ t = \frac{\log 2}{4 \log 1.02} = 8.75 \]
6. For this graph, give the equations of the:
   a) vertical asymptotes;
   b) horizontal asymptotes.

\[ a) \quad x = -2, \quad x = 2 \]

\[ b) \quad y = 0; \quad y = 20 \]

7. A firm owns 10 machines, each of which can produce 40 widgets per hour. It costs $25 to set up each machine. A supervisor to oversee the operation of all the machines earns $20 per hour. How many machines should be set up in order to minimize the cost of producing 10,000 widgets?

\[ q = \text{# of machines}; \quad 25q = \text{setup cost}; \quad \frac{10,000}{40q} = 250/q = \text{time} \]

\[ \frac{5000}{q} = \text{cost for supervisor} \]

\[ C(q) = 25q + \frac{5000}{q} \quad \text{for } 0 \leq q \leq 10 \]

\[ C'(q) = 25 - \frac{5000}{q^2} = 0 \quad \text{for } q = \sqrt{200} = 10\sqrt{2} > 10 \]

\[ C'(q) < 0 \quad \text{for } 0 \leq q \leq 10. \quad \text{So set up 10 machines} \]

8. Given that the population \( t \) years from now will be \( P(t) = 50/(1 + 10 e^{-0.1t}) \), find:
   a) the population 10 years from now; b) the rate at which the population will be changing with respect to time 10 years from now.

\[ a) \quad P(10) = 50/(1 + 10 e^{-1}) = 10.69 \]

\[ b) \quad P'(t) = -50/(1 + 10 e^{-0.1t})^2 (-10 \cdot 0.1 e^{-0.1t}) \]

\[ P'(10) = -50/(1 + 10 e^{-1})^2 (-e^{-1}) \]

\[ = 84 \]