4.3. Differentiation of Logarithmic and Exponential Functions

Derivative of $\ln x$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \text{for} \quad x > 0$$

Example

Differentiate the function $f(x) = x \ln \sqrt{x}$. 
Differentiation of Logarithmic Functions

The Chain Rule for Logarithmic Functions

If $u(x)$ is a differentiable function of $x$, then

$$
\frac{d}{dx}[\ln u(x)] = \frac{u'(x)}{u(x)}
$$

Example

Differentiate the function $f(x) = \ln(x^2 + 1)$.
Differentiation of Logarithmic Functions

Example
Differentiate the function $f(x) = \ln(x^3 - 5x + 4)$. 
Example
Find an equation for the tangent line to $y = x + \ln x$ at the point where $x = e$. 
Differentiation of Exponential Functions

The Derivative of the Exponential Function

\[ \frac{d}{dx}(e^x) = e^x \quad \text{for every real number } x \]

Example

Differentiate the function \( f(x) = \frac{e^x}{x} \).
The Chain Rule for Exponential Functions

If \( u(x) \) is a differentiable function of \( x \), then

\[
\frac{d}{dx} e^{u(x)} = e^{u(x)} u'(x)
\]

Example

Differentiate the function \( f(x) = xe^{2x} \).
Example

Find the largest and smallest values of the function $F(x) = e^{x^2-2x}$ over the closed interval $0 \leq x \leq 2$. 
Logarithmic Differentiation

Differentiating a function that involves products, quotients, or powers can often be simplified by first taking the logarithm of the function.

**Step 1.** Take logarithms of both sides of the expression for \( f(x) \) and simplify the resulting equation.

**Step 2.** Use the chain rule to differentiate both sides.

**Step 3.** Multiply both sides with \( f(x) \) to get \( f'(x) \).
Logarithmic Differentiation

Example

Use logarithmic differentiation to find the derivative of

\[ f(x) = \sqrt[4]{\frac{2x + 1}{1 - 3x}}. \]
Logarithmic Differentiation

Example

Use logarithmic differentiation to find the derivative of

\[ f(x) = \frac{e^{3x}(x^2 + 5)}{(1 - x)^5}. \]