Absolute Maxima and Minima of a function
Let $f$ be a function defined on an interval $I$ containing the number $c$. Then

- $f(c)$ is the **absolute maximum** of $f$ on $I$ if $f(c) \geq f(x)$ for all $x$ in $I$.

- $f(c)$ is the **absolute minimum** of $f$ on $I$ if $f(c) \leq f(x)$ for all $x$ in $I$.

Collectively, absolute maxima and minima are called **absolute extrema**.
The Extreme Value Property
A function $f(x)$ that is continuous on the closed interval $a \leq x \leq b$ attains its absolute extrema on the interval either at an endpoint of the interval ($a$ or $b$) or at a critical number $c$ such that $a < c < b$.

How to Find the Absolute Extrema of a Continuous Function $f$ on $a \leq x \leq b$

Step 1. Find all critical numbers of $f$ in $a < x < b$.

Step 2. Compute $f(x)$ at the critical numbers found in step 1 and at the endpoints $x = a$ and $x = b$.

Step 3. The largest and smallest values found in step 2 are, respectively, the absolute maximum and absolute minimum values of $f(x)$ on $a \leq x \leq b$. 
Example
Find the absolute maximum and absolute minimum (if any) of

\[ f(x) = x^3 + 3x^2 + 1; \quad -3 \leq x \leq 2. \]
Example
Find the absolute maximum and absolute minimum (if any) of

\[ f(t) = \frac{t^2}{t - 1}; \quad -2 \leq t \leq 1. \]
Absolute Extrema on a general interval

Example
Find the absolute maximum and absolute minimum (if any) of

\[ f(u) = u + \frac{16}{u}; \quad u > 0. \]
Absolute Extrema on a general interval

The Second Derivative Test for Absolute Extrema
Suppose that $f(x)$ is continuous on $I$ where $x = c$ is the only critical number and that $f'(c) = 0$. Then

- if $f''(x) > 0$, the absolute minimum of $f(x)$ on $I$ is $f(c)$.
- if $f''(x) < 0$, the absolute maximum of $f(x)$ on $I$ is $f(c)$.

Example
Find the absolute maximum and absolute minimum (if any) of

$$f(u) = u + \frac{16}{u}; \quad u > 0.$$