The Product Rule
If \( f(x) \) and \( g(x) \) are differentiable at \( x \), then so is their product and
\[
\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]
\]
or equivalently
\[
(fg)' = fg' + gf'
\]

Example
Differentiate \( f(x) = (2x - 5)(1 - x) \).
The Product Rule

Example
Differentiate $f(x) = (x^3 - 2x^2 + 5)(\sqrt{x} + 2x)$. 
The Quotient Rule

If $f(x)$ and $g(x)$ are differentiable functions, then so is the quotient $Q(x) = \frac{f(x)}{g(x)}$ and

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx}[f(x)] - f(x) \frac{d}{dx}[g(x)]}{g^2(x)}$$

or equivalently

$$\left( \frac{f}{g} \right) = \frac{gf' - fg'}{g^2}$$

Example

Differentiate $y = \frac{1 + x^2}{1 - x^2}$. 
The Quotient Rule

Example

Find all points on the graph of $f(x) = \frac{x^2 + x - 1}{x^2 - x + 1}$ where the tangent line is horizontal.
Example

Differentiate \( g(x) = \frac{(x^2 + x + 1)(4 - x)}{2x - 1} \).
The Second Derivative

The second derivative of a function is the derivative of its derivative. If \( y = f(x) \), the second derivative is denoted by

\[
\frac{d^2y}{dx^2} \quad \text{or} \quad f''(x)
\]

The second derivative gives the rate of change of the rate of change of the original function.

Example
Find the second derivative of \( f(x) = x^{10} - 4x^6 - 27x + 4 \).
The Second Derivative

Example

Find the second derivative of \( y = (x^2 - 2x) \left( x - \frac{1}{x} \right) \).
The $n$th Derivative
For any positive integer $n$, the $n$th derivative of a function is obtained from the function by differentiating successively $n$ times. If the original function is $y = f(x)$, the $n$th derivative is denoted by

$$\frac{d^n y}{dx^n} \quad \text{or} \quad f^{(n)}(x)$$

Example
Find $f^{(4)}(x)$ if $f(x) = x^6 - 2x^5 + x^4 - 3x^3 + 5x - 6$. 