2.2. Techniques of Differentiation

The Constant Rule
For any constant \( c \), \( \frac{d}{dx}[c] = 0 \)

The Power Rule
For any real number \( n \), \( \frac{d}{dx}[x^n] = nx^{n-1} \)

Example
Differentiate the function \( y = \sqrt{x^5} \).
The Constant Multiple Rule

If $c$ is a constant and $f(x)$ is differentiable, then so is $cf(x)$ and

$$\frac{d}{dx}[cf(x)] = c \frac{d}{dx}[f(x)]$$

Example
Differentiate the function $y = 2^{\frac{3}{\sqrt{x^4}}}$. 
The Sum Rule

If \( f(x) \) and \( g(x) \) are differentiable, then so is their sum and

\[
\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]
\]

Example

Differentiate the function \( y = \frac{2}{x} - \frac{2}{x^2} + \frac{1}{3x^3} \).
Differentiation of polynomials

Example
Differentiate the function $y = x^3(x^2 - 5x + 7)$. 
Example

Find the equation of the line that is tangent to the graph of the function \( y = \sqrt{x^3 - x^2 + \frac{16}{x^2}} \) at the point \((4, -9)\).
The relative rate of change of a quantity $Q(x)$ with respect to $x$ is

$$\frac{Q'(x)}{Q(x)}$$

The corresponding percentage rate of change of $Q(x)$ with respect to $x$ is

$$\frac{100Q'(x)}{Q(x)}$$
Example

It is estimated that $t$ years from now, the population of a certain town will be $P(t) = t^2 + 100t + 8,000$.

a. Express the percentage rate of change of the population as a function of $t$.

b. What will happen to the percentage rate of change of the population in the long run?
Rectilinear Motion

Motion of an object along a line is called rectilinear motion. If the position at time $t$ of an object moving along a straight line is given by $s(t)$, the object has

velocity $v(t) = s'(t) = \frac{dx}{dt}$

and

acceleration $a(t) = v'(t) = \frac{dv}{dt}$.

The object is advancing when $v(t) > 0$, retreating when $v(t) < 0$, and stationary when $v(t) = 0$. It is accelerating when $a(t) > 0$ and decelerating when $a(t) < 0$. 

Example

The position at time $t$ of an object moving along a line is given by $s(t) = t^3 - 9t^2 + 15t + 25$.

a. Find the velocity of the object.

b. Find the total distance traveled by the object between $t = 0$ and $t = 6$.

c. Find the acceleration of the object and determine when the object is accelerating and decelerating between $t = 0$ and $t = 6$. 