1 Decide if the following function is continuous at \( x = 2 \). Explain the reason.

\[
    f(x) = \begin{cases} 
    x^2 - x - 2 & \text{if } x < 2 \\
    x^2 - 3x + 2 & \text{if } x \geq 2 
    \end{cases}
\]

**Solution.** We need to verify the three criteria for continuity are satisfied.

(a) \( f(x) \) is defined; \( f(x) = 2^2 - 2 = 2 \).

(b) To decide whether the limit at \( x = 2 \) exists, we need to find two one-sided limits;

\[
    \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2 - x - 2}{x^2 - 3x + 2} = \lim_{x \to 2^-} \frac{(x - 2)(x + 1)}{x - 1} = \lim_{x \to 2^-} \frac{x + 1}{x - 1} = \frac{3}{1} = 3
    
    \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 - x) = 2^2 - 2 = 2
\]

Since two one-sided limits have different values, \( \lim_{x \to 2} f(x) \) does not exist.

Therefore, \( f(x) \) is not continuous at \( x = 2 \).

2 Find the slope of the tangent line to the curve \( y = 2\sqrt{x^3} + \frac{1}{x^2} \) at the point where \( x = 1 \).

**Solution.** The derivative of the given function with respect to \( x \) is

\[
\frac{dy}{dx} = \frac{d}{dx} \left[ 2x^{3/2} + x^{-2} \right] = 2 \frac{d}{dx} \left[ x^{3/2} \right] + \frac{d}{dx} \left[ x^{-2} \right] = 2 \left( \frac{3}{2} \right) x^{1/2} + (-2)x^{-3} = 3\sqrt{x} - \frac{2}{x^3}
\]

Thus the slope of the tangent line to the given curve where \( x = 1 \) is given by

\[
\left. \frac{dy}{dx} \right|_{x=1} = 3\sqrt{1} - \frac{2}{1^3} = 1
\]