1. [15] Suppose $A$ is an arbitrary $5 \times 2$ matrix.
   
   (a) Consider the elementary row operation that replaces row 4 with itself plus $2 \times$ Row 1. What is the corresponding elementary matrix $E$?
   
   (b) Consider the elementary row operation that interchanges rows 2 and 5. What is the corresponding elementary matrix $F$?
   
   (c) Consider the elementary row operation that scales row 2 by $\frac{1}{2}$. What is the corresponding elementary matrix $G$?

2. [20] Suppose $A$, $B$, $C$, $D$, and $X$ are square $n \times n$ matrices and that satisfy the equation $A + BX = C + DX$.
   
   (a) What matrix (some combination of $A$, $B$, $C$, and $D$) needs to be invertible in order that we can solve this equation for $X$?
   
   (b) Assuming the matrix from part (a) is invertible, solve for $X$.

3. [30 pts] Assume that $A = [a_1 \ a_2 \ a_3 \ a_4 \ a_5]$ and $B = [b_1 \ b_2 \ b_3 \ b_4 \ b_5]$ are row equivalent, where

   $$\begin{bmatrix}
   1 & 2 & -2 & 0 & 7 \\
   -2 & -3 & 1 & -1 & -5 \\
   -3 & -4 & 0 & -2 & -3 \\
   3 & 6 & -6 & 5 & 1
   \end{bmatrix}, \quad
   \begin{bmatrix}
   1 & 0 & 4 & 0 & -3 \\
   0 & 1 & -3 & 0 & 5 \\
   0 & 0 & 0 & 1 & -4 \\
   0 & 0 & 0 & 0 & 0
   \end{bmatrix}$$

   (a) Determine a basis for Col $A$.
   
   (b) Determine a basis for Row $A$.
   
   (c) Determine rank $A$.
   
   (d) Determine $\dim \text{Nul } A$.
   
   (e) Fill in the blanks: Nul $A$ is a _____ dimensional subspace of $\mathbb{R}^k$, where $k =$ _____.

4. [30] Let $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ be bases for $\mathbb{R}^2$, where

   $$b_1 = \begin{bmatrix}
   1 \\
   4
   \end{bmatrix}, \quad
   b_2 = \begin{bmatrix}
   1 \\
   1
   \end{bmatrix}, \quad
   c_1 = \begin{bmatrix}
   -1 \\
   8
   \end{bmatrix}, \quad
   c_2 = \begin{bmatrix}
   1 \\
   -5
   \end{bmatrix}$$

   (a) Calculate the change-of-coordinates matrix from $B$ to $C$.
   
   (b) If $[x]_C = \begin{bmatrix}
   1 \\
   -1
   \end{bmatrix}$
      
      i. Determine $[x]_B$.
      
      ii. Determine $x$.

5. [20] Let $H$ denote the subset of $\mathbb{P}_3$ (the vector space of all polynomials of degree at most 3) consisting of all polynomials of the form $p(t) = a + (a + b)t + bt^2$, where $a$, $b \in \mathbb{R}$.
   
   (a) Show that $H$ is a subspace of $\mathbb{P}_3$.
   
   (b) Determine two vectors that span $H$. 