1. [20] Calculate the general solution of the following system.

\[
\begin{align*}
5x_1 - 9x_2 + 18x_4 &= -7 \\
3x_2 - 4x_4 &= 2 \\
6x_2 - 8x_4 &= 4
\end{align*}
\]

2. [15] For each matrix below determine whether its columns form a linearly independent set. Give reasons for your answers. (Make as few calculations as possible.)

\[
A = \begin{bmatrix}
-4 & 12 \\
1 & -3 \\
-3 & 8
\end{bmatrix}, \quad B = \begin{bmatrix}
2 & 7 & 0 \\
-4 & -6 & 5 \\
6 & 13 & -3
\end{bmatrix}, \quad C = \begin{bmatrix}
1 & 5 & -3 & 2 \\
0 & 4 & -9 & 18 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

3. [15] For each matrix in Problem 2 determine if the columns of the matrix span \(\mathbb{R}^3\). Give reasons for your answers. (Again, make as few calculations as possible.)

4. [15] Let \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) be a linear transformation such that

\[
T(e_1) = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \quad T(e_2) = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \quad T(e_3) = \begin{bmatrix} 0 \\ -7 \\ 8 \end{bmatrix}
\]

(a) Determine the standard matrix of \( T \).
(b) Determine if \( T \) is a one-to-one transformation. Mention an appropriate theorem/result to justify your answer.
(c) Provide a formula for \( T(x_1, x_2, x_3) \).

5. [15] Determine the standard matrix of the linear transformation \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) that reflects points through the line \( x_2 = x_1 \) and then reflects the result through the line \( x_2 = 0 \).

6. [15 pts] Consider the linear system \( Ax = b \), where

\[
A = \begin{bmatrix}
1 & 0 & 0 \\
2 & 0 & 1 \\
0 & -1 & 0
\end{bmatrix}, \quad b = \begin{bmatrix}
1 \\
1 \\
-1
\end{bmatrix}
\]

(a) Show that \( A \) is invertible
(b) Calculate \( A^{-1} \).
(c) Use part (b) to solve \( Ax = b \)

7. [15] Mark each statement as True (T) or False (F). You do not have to justify your answer.

(a) In some cases it is possible for six vectors to span \( \mathbb{R}^5 \).
(b) If \( A \) is an \( m \times n \) matrix and if the equation \( Ax = b \) has a solution for some \( b \), then the columns of \( A \) span \( \mathbb{R}^m \).
(c) If a system of linear equations has two different solutions, then it has infinitely many solutions.
(d) Every matrix is row equivalent to a unique matrix in echelon form.
(e) If \( v_1 \) and \( v_2 \) span a plane in \( \mathbb{R}^3 \) and if \( v_3 \) is not in that plane, then \( \{v_1, v_2, v_3\} \) is a linearly independent set.