Thin Fluid Films over Thin Porous Layers

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What are Thin Fluid Films?

- Thin fluid films are:
  - Tear film on the exterior surface of a contact lens
  - Fluid layers thickness is much less than the lateral extent
  - Nanometer to micrometer in thickness
- Another is the contact lens that mathematically is represented as porous layers (ref. Raad and Sabau)

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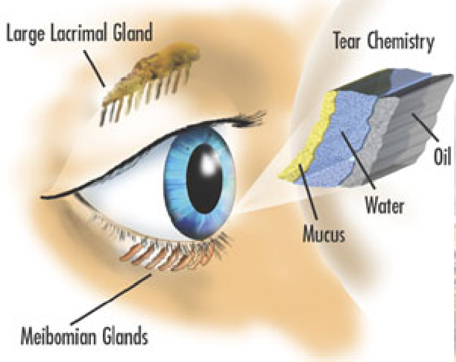
***** Goal of the Study *****

- The most common disease, **Dry Eye Sydrome** in eye care industry
- **Dry Contact Lens** that causes irritation on the eye.

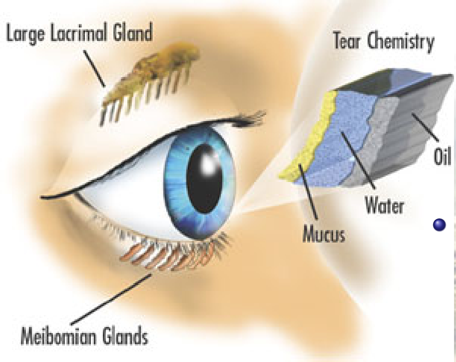
Image courtesy of: http://www.emedicinehealth.com

Introduction to Tear Films



- Fluid Layers on the eye
  - Oil Layer
    60.3+/- 2.5 nm [1]
  - Watery Layer ('pre-lens' film)
    4 - 7μm [2]
  - Mucus Layer
    2 - 7μm [2]

Image courtesy of:
http://www.naturecoasteye.com



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  - Mucus Layer
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- Porous layers
  - Contact lens
    average 30μm [3]

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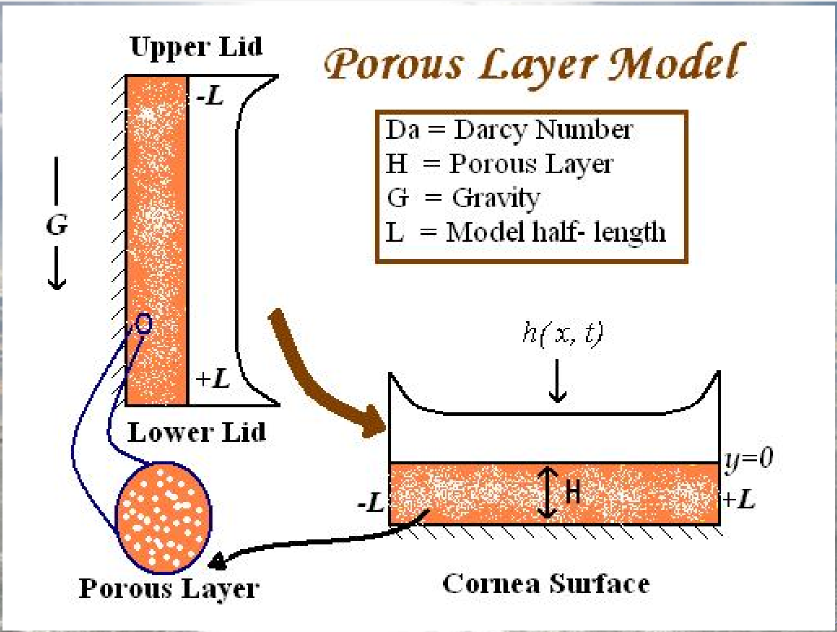
References:
1. D.Sullivan, D. Dartt, M. Meneray
2. Nichols, Chiappino, Dowson, M.D
3. P.E. Raad and A.S. Sabau

Construct Dry Eye Model



Introduction to Thin Films

- Identify the behavior of thin flud films with respect to time
- Factors: gravity, pressure of a porous layer, the presence of a porous layer underneath the thin liquid film and more...

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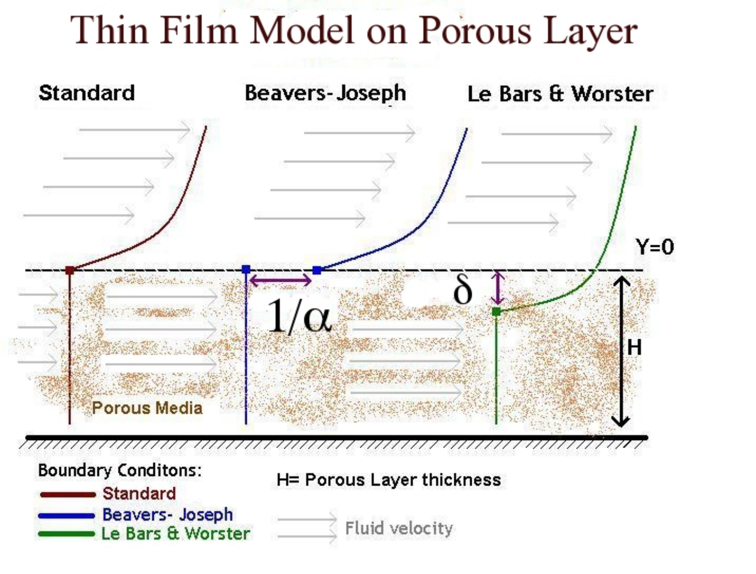
NOTE:

We examine a modification of the Braun & Fitt (BF) Equation

- Standard Case :
  ** non-slip on the pore boundary
- Beavers - Joseph Boundary Condition Case (BJ) :
  ** slip on the pore boundary
- Le Bars & Worster Boundary Condition Case (LW) :
  ** slip into the pore boundary



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Thin Film Evolution Equations

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}\left\{ f\left(h\right) * \left(\frac{1}{Ca}\frac{\partial^3 h}{\partial x^3} - Gy\cos\theta\frac{\partial h}{\partial x} + Gx\sin\theta \right) \right\}.$$

*** Note: Evaporation effect is not included ***

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- Le Bars & Worster

$$f_{LW} = \frac{(h+\delta)^3}{3} + Da(h+H)$$

Interests

Specific cases

1. Verification method, we compare data to the result of BF equation

*by set Da= 0, and $1/\alpha$ or $\delta = 0$

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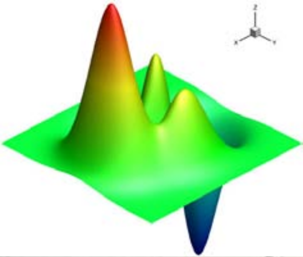
4. The gravity effect for $G = \frac{1}{4}$
Specifically at $\theta = \frac{\pi}{2}$ (up-right posistion)

Numerical Method



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  - Spatial Derivatives Representation
  - Half-step scheme for third and forth order derivative

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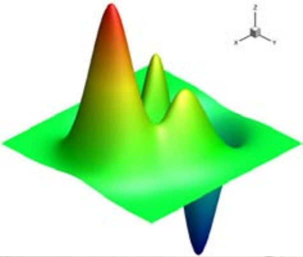

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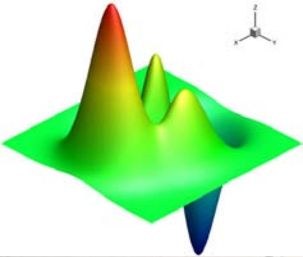
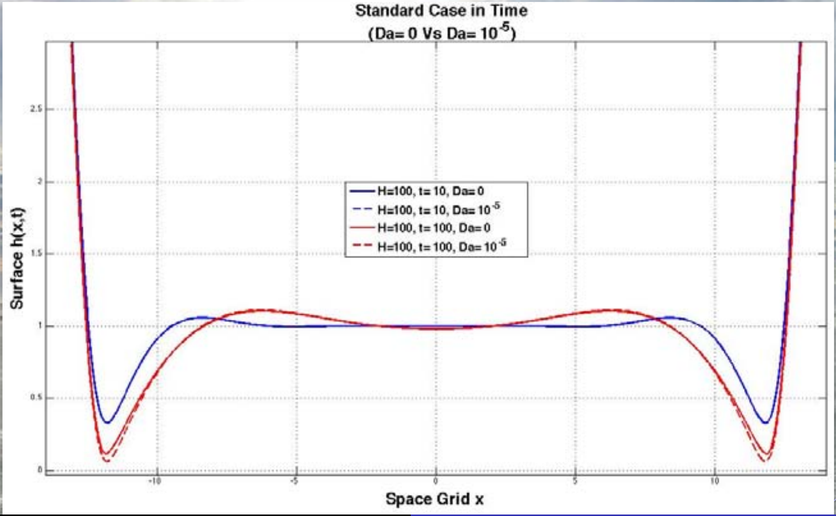


- Finite difference method
  - Spatial Derivatives Representation
  - Half-step scheme for third and forth order derivative
- Taylor series
  - One sided-derivative approximation - boundary condition
- Matlab Simulation
  - ODE solvers - ode23s, ode45

Main Result

- Standard case vs. Braun Fitt Study
  - Time increases when Da= 0 vs. Da= 10^{-5}, H= 100, and G= 0



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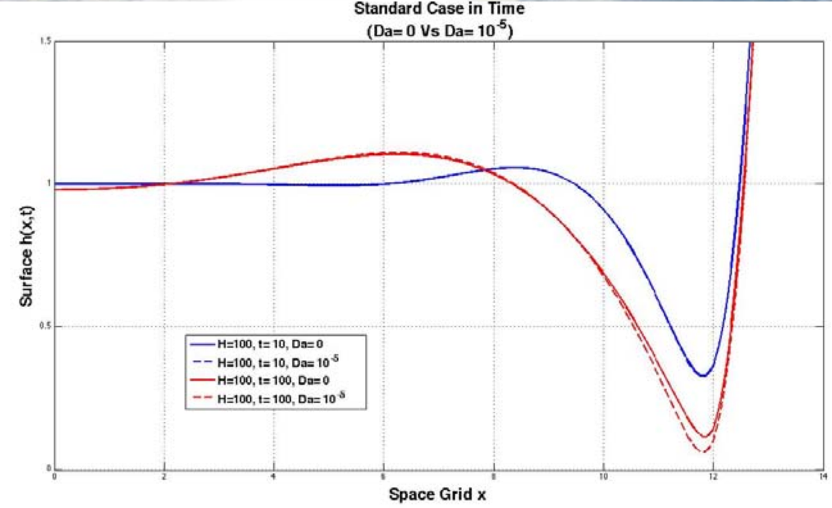
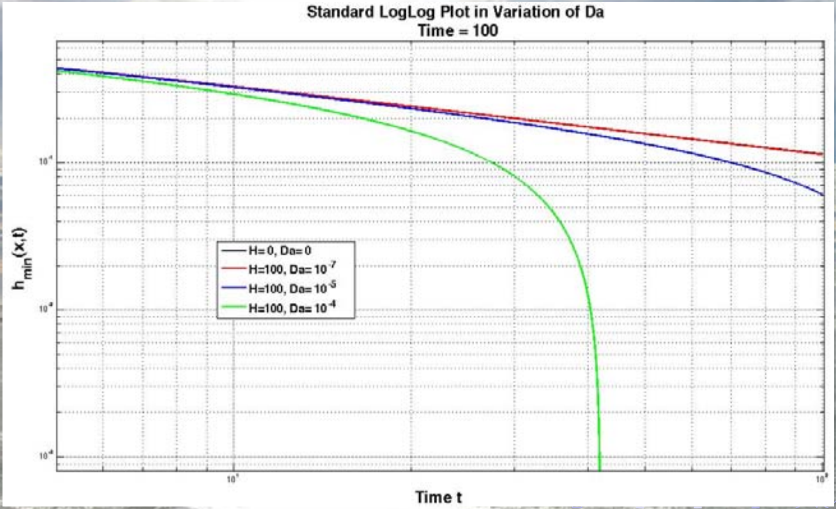


Main Result - con't

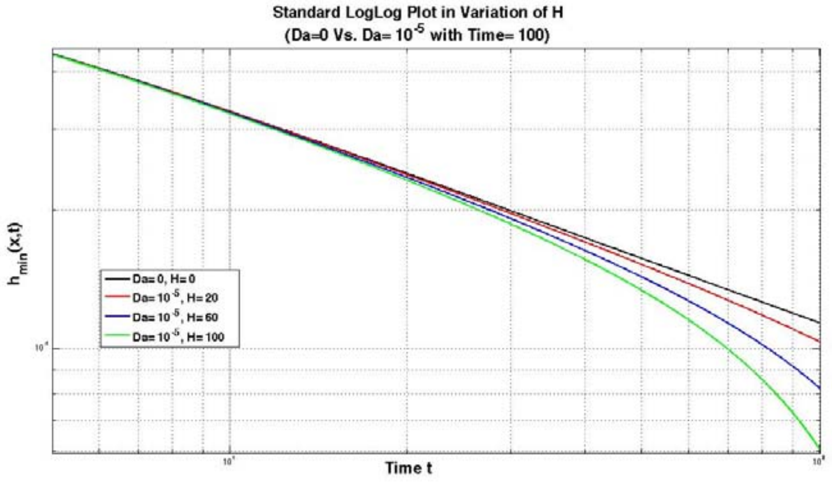
- This movie is demonstrate the behavior of the surface of the thin fluid film over time (t $=100$).

Main Result - con't

- Standard Boundary Condition
  - Variation of Da and H= 100

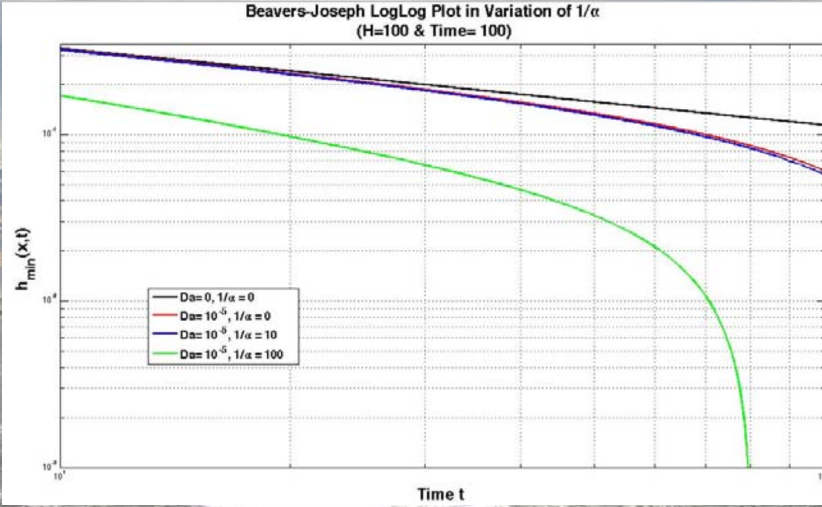


- Standard Boundary Condition
  - Variation of H and Da $= 10^{-5}$



Main Result - con't

- Beavers-Joseph Boundary Condition
  - Variation $1/\alpha$, Da$= 10^{-5}$ and H$= 100$



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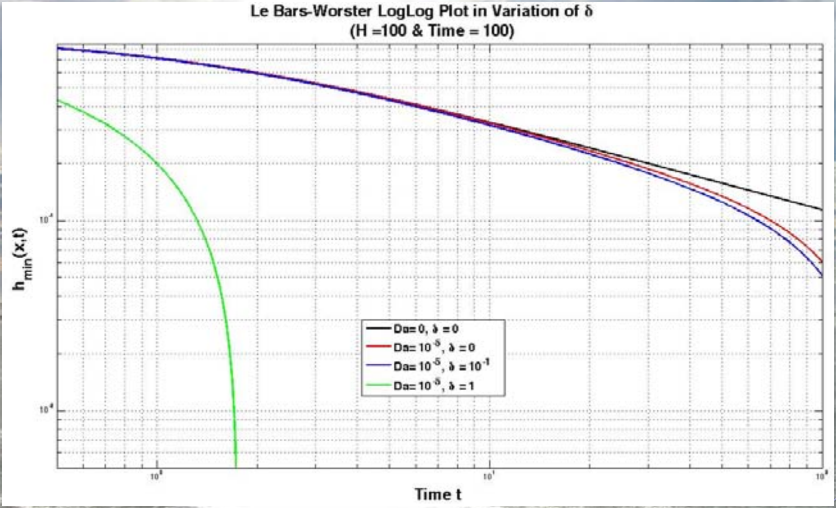
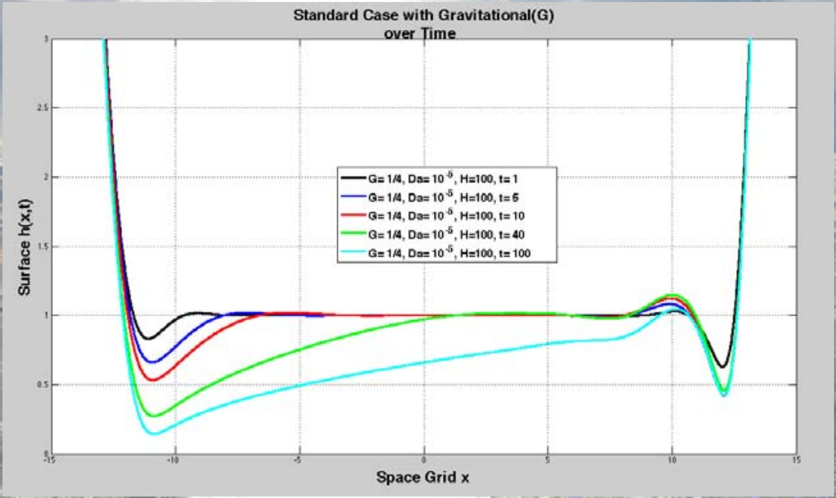
- Le Bars & Worster Boundary Condition
  - Variation δ, Da$= 10^{-5}$, and H$= 100$



Main Result - con't

- Standard case
  $G = \frac{1}{4}$ when time t is vary



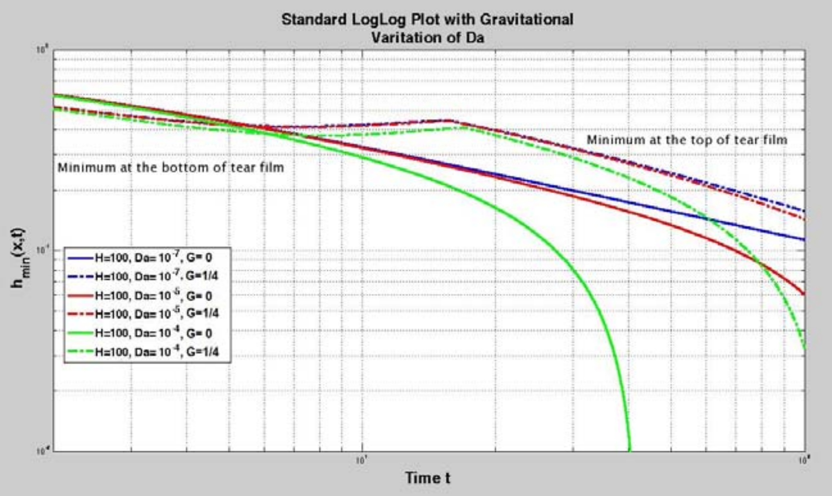
Main Result - con't

The movie is demonstrate the effect of gravity $G = \frac{1}{4}$ at $\theta = \frac{\pi}{2}$

- Da= 10^{-5}, H= 100, and t = 100

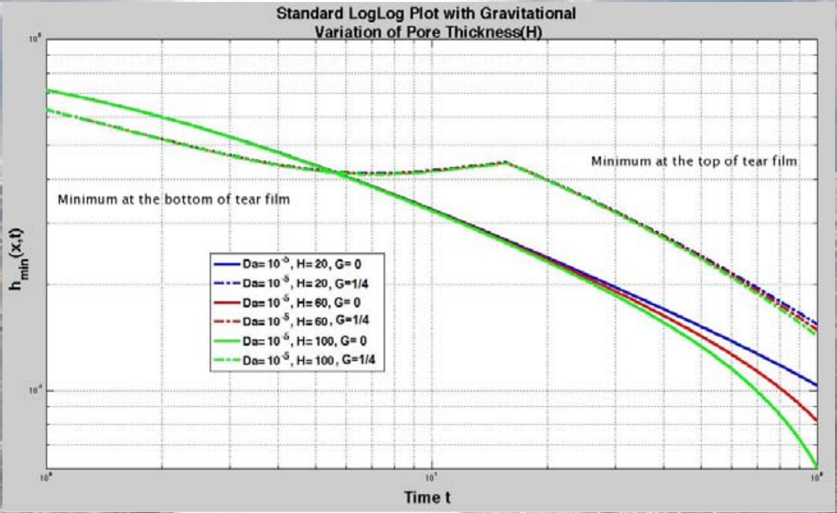
Main Result - con't

- Take a closer look when we compare it to $G = 0$ and vary Da



Main Result - con't

- Now, we compare it to $G = 0$ and vary H



For all equations, the simulated models of the films start to **RUPTURE** as **ANY** of the following happen:

- time **increased**

Discussion

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  Note : ratio of pore scale radius of contact lens to the fluid film thickness

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- the gravity(G) is slower the rate of film rupture.

Conclusion

Why do these models help people like you and me?

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- The gravity increases the lifetime of the film, but all other effects still play a major role on rupture of the film.
- Using the right properties of contact lens can maximize the lifetime of the fluid films.

Image courtesy of www.targetwoman.com

Future Works

- Improve the numerical scheme, using Spectral Method
- May also convert to FORTRAN code for faster computation
- Optimization on the effect of variables to the realistic value

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Thank You

- Dr. Daniel M. Anderson
- GMU-URCM Team and Staff
- NSF - CSUMS Program grant NSF DMS-0639300.
- GMU - Mathematics Department
- College of William and Mary

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