Self Organization in a Diffusion Model of Thin Electric Current Sheets

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Introduction

Upcoming Topics

- Space Weather
  - Solar Wind
  - Earth's Magnetosphere

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- Self Organized Criticality
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- Analysis
  - Model
  - Total Field Energy
  - Time Averaged Mean
  - Power Spectral Density

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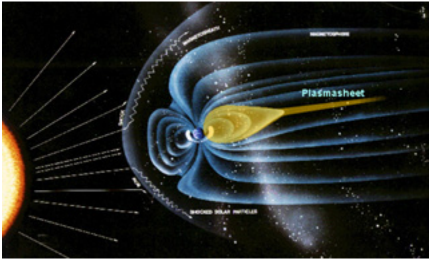
- Conclusions
- Future Work

Space Weather



- Solar Wind
  - Space Plasma

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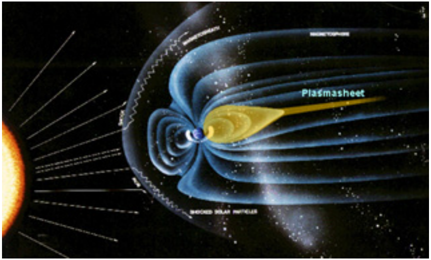

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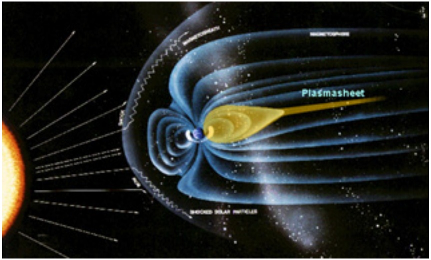
- Magnetosphere
  - Region in space, surrounding the earth, composed of charged particles and governed by magnetic flux.

Space Weather

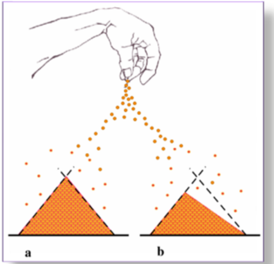


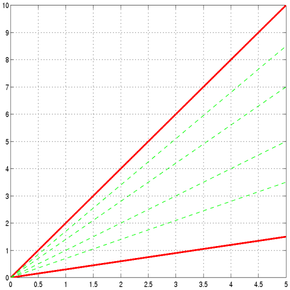
- Solar Wind
  - Space Plasma
- Magnetosphere
  - Region in space, surrounding the earth, composed of charged particles and governed by magnetic flux.
- Plasma Sheet
  - Site of Reconnection

Goals

- Describe the physical interactions within the system:
  - Lead to better predictions when forecasting space weather
  - Aid Development of a Physical Theory for this system

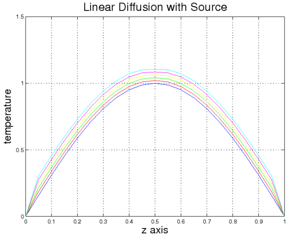
Self Organized Criticality



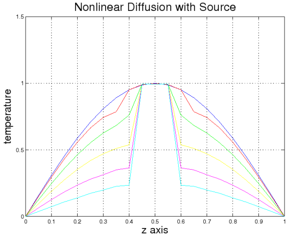


- Simply rules govern the dynamics
- Thresholds exist within the system
- The threshold is eventually exceeded by the build up of energy
- Systems displaying characteristics associated with SOC dissipate stored energy in avalanches [3].

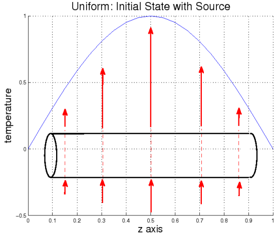
Instability



$$\frac{\partial}{\partial t}B(z,t) = D_{min}\frac{\partial^2}{\partial z^2}B(z,t) + S(z)$$



$$\frac{\partial}{\partial t}B(z,t) = \frac{\partial}{\partial z}\left(D(z,t)\frac{\partial}{\partial z}B(z,t)\right) + S(z)$$



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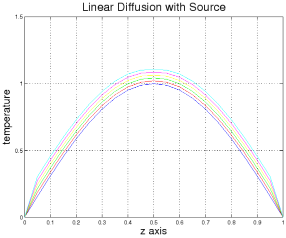

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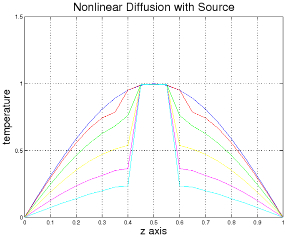

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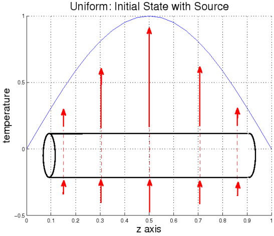
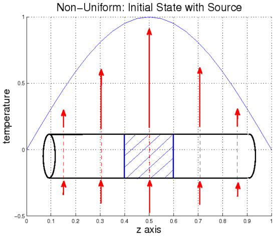
- 

- 

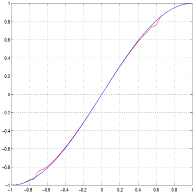
Model Description

The Model is Given by:

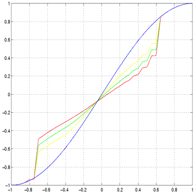
$$\frac{\partial B_x}{\partial t} = \frac{\partial}{\partial z}\left(D(z,t)\frac{\partial B_x}{\partial z}\right) + S(z) \tag{1}$$

$$\frac{\partial}{\partial t}\left(D(z,t)\right) = \frac{Q\left(\left|\frac{\partial B_x}{\partial z}\right|\right)}{\tau} - \frac{D}{\tau} \tag{2}$$

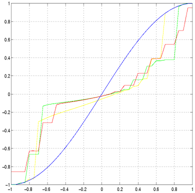
$$Q\left(\left|\frac{\partial B_x}{\partial z}\right|\right) = \begin{cases} D_{min} & \text{for low state} \\ D_{max} & \text{for high state} \end{cases} \tag{3}$$



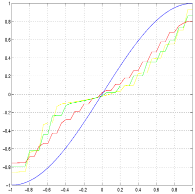
Time 1



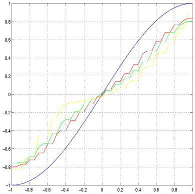
Time 2



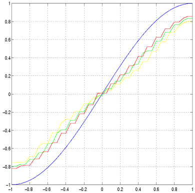
Time 3



Time 4

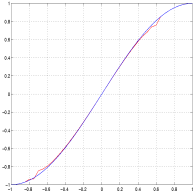


Time 5



Time 6

- The input of energy by the source term drives the system to the point of criticality



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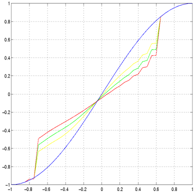

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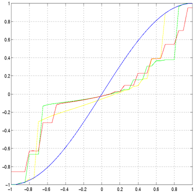

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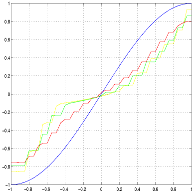

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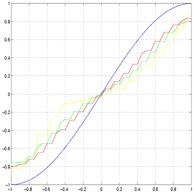

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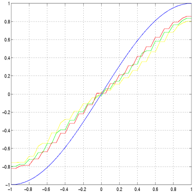

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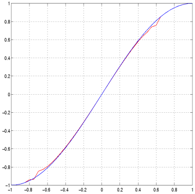
- The input of energy by the source term drives the system to the point of criticality
- Once the critical point is reached, the system reacts by unloading the energy in avalanches



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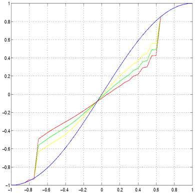

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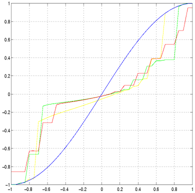

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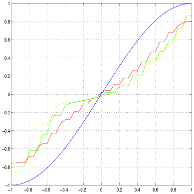

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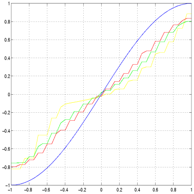

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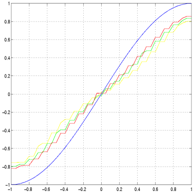

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- The input of energy by the source term drives the system to the point of criticality
- Once the critical point is reached, the system reacts by unloading the energy in avalanches
- System returns to a stable state, but steep slopes will be present in many local spatial positions [3].

Total Energy

The total energy of the system at any instant is defined:

$$E(t) = \int (B_x)^2 \, dz$$

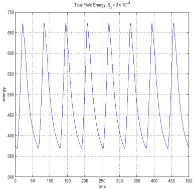


Figure 1: $S_0 = 3 \times 10^{-4}$

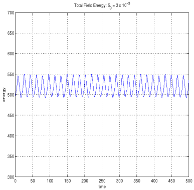


Figure 2: $S_0 = 10^{-4}$

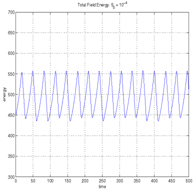


Figure 3: $S_0 = 3 \times 10^{-3}$

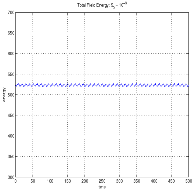
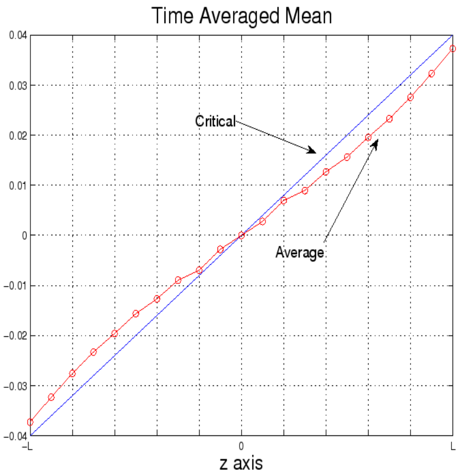


Figure 4: $S_0 = 10^{-3}$

Time Averaged Mean



- Field strength balanced by induced dynamic state
- System remains close to, but under the critical state

Parameters

- Free Parameters:
  - τ, D_{min}, D_{max}, S_0, k, β

Parameters

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- Reduction in the number of parameters
  - To analyze the effects of the free parameters on the systems ability to attain an SOC state
  - D_r, S_0', β
  - Where D_r is the ratio of D_{min} and D_{max}, i.e $D_r = \frac{D_{min}}{D_{max}}$

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  - Where D_r is the ratio of D_{min} and D_{max}, i.e $D_r = \frac{D_{min}}{D_{max}}$
- The system becomes:

$$\frac{\partial B_x'}{\partial t'} = \frac{\partial}{\partial z'}\left(D(z',t')\frac{\partial B_x'}{\partial z'}\right) + S_0' sin\left(\frac{\pi z'}{2L'}\right) \tag{4}$$

$$\frac{\partial}{\partial t'}\left(D'(z',t')\right) = Q'\left(\left|\frac{\partial B_x'}{\partial z'}\right|\right) - D' \tag{5}$$

$$Q'\left(\left|\frac{\partial B_x'}{\partial z'}\right|\right) = \begin{cases} D_r & \text{low state} \\ 1 & \text{high state} \end{cases} \tag{6}$$

Parameters

- Free Parameters:
  - τ, D_{min}, D_{max}, S_0, k, β
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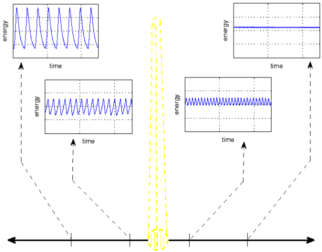
  - To analyze the effects of the free parameters on the systems ability to attain an SOC state
  - D_r, S'_0, β
  - Where D_r is the ratio of D_{min} and D_{max}, i.e $D_r = \frac{D_{min}}{D_{max}}$
- The system becomes:

$$\frac{\partial B'_x}{\partial t'} = \frac{\partial}{\partial z'}\left(D(z',t')\frac{\partial B'_x}{\partial z'}\right) + S'_0 sin\left(\frac{\pi z'}{2L'}\right) \quad (4)$$

$$\frac{\partial}{\partial t'}\left(D'(z',t')\right) = Q'\left(\left|\frac{\partial B'_x}{\partial z'}\right|\right) - D' \quad (5)$$

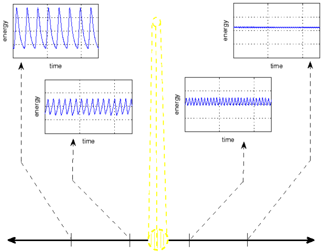
$$Q'\left(\left|\frac{\partial B'_x}{\partial z'}\right|\right) = \begin{cases} D_r & \text{low state} \\ 1 & \text{high state} \end{cases} \quad (6)$$

Analysis: Nondimensional Model



- Values of remaining parameters:

Analysis: Nondimensional Model



- Values of remaining parameters:
    - D_r = fixed, β = fixed, S'_0 = varied

Movie (click to play)

Conclusions and Future Work

- The Diffusive System Displays Characteristics associated with Self-Organized Criticality
- The number of states the system takes on is much greater then what was previously known.
- Need to research/develop a smarter algorithm for analysis.

Thanks

- Special thanks to...
  - Dr. Robert Weigel
  - URCM
  - Lu and Klimas, for their prior work
  - and of course, the audience

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