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An investigation of student discovery of the concept of eigenvector in the context of 2-D linear vector fields.

Robert Sachs

Department of Mathematical Sciences
George Mason University
Fairfax, Virginia 22030

`rsachs@gmu.edu`

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- Genesis of the idea – teaching multivariable calculus, motivating linear algebra study

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- Constrained min/max for quadratic functions, tied to classifying unconstrained critical points
- Know that gradient fields (linear) come from symmetric matrices
- Built a basic Mathematica demonstration for displaying linear gradient fields in 2-D with sliders

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- Teach ODEs where we use eigenvectors, eigenvalues in linear systems
- Working on multivariable calculus book and want to understand how students view different visualizations

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- Fourth question: do gradient fields have circulation?

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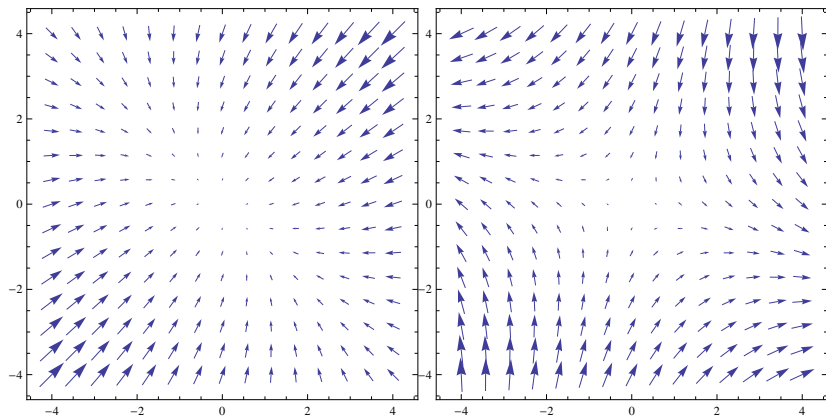
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- use of frames vs. static pictures

Go to Mathematica hopefully

Here is where we go to Mathematica if laptop hookup is working!!

Some visualizations



- For linear vector fields, have in general two lines that divide positive and negative components (think of velocity field)

Proving existence via qualitative methods

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- Symmetric part has special property of how these relate to usual four quadrants
- Must have overlap in first quadrant between radial field and vector field either in “quadrant” I or III (positive or negative eigenvalue respectively)
- Same in second quadrant of radial field – distinct from having quadrant I for radial tied to quadrant II for vector field!

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- Splitting general vector field as sum of symmetric and skew parts

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- Usual gradient flow lines from odes

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- Happy to work on examples – conjectured first derivative test and a corresponding level set test
- Balancing act between leading questions and open exploration
- Plan on using more advanced undergraduates to build and evaluate some of these environments
- Visual aspects are quite compelling and easy to generate